

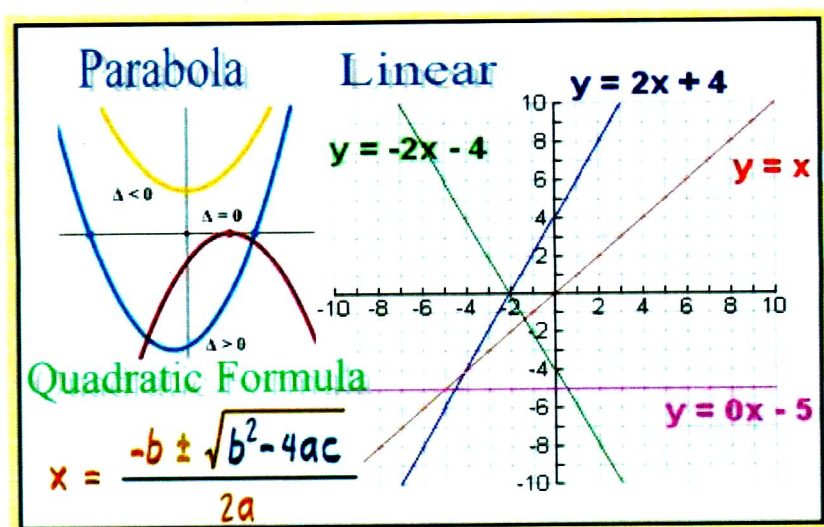


DEPARTMENT OF EDUCATION

GRADE 10

MATHEMATICS

UNIT 4



FUNCTIONS AND GRAPHS

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2016

GRADE 10

MATHEMATICS

UNIT 4

FUNCTIONS AND GRAPHS

TOPIC 1: STATISTICAL GRAPHS

**TOPIC 2: LINEAR OR STRAIGHT LINE
GRAPHS**

**TOPIC 3: QUADRATIC FUNCTIONS
AND PARABOLA**

TOPIC 4: SYSTEMS OF EQUATIONS

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MR. DEMAS TONGOGO
Principal - FODE



Flexible Open and Distance Education
Papua New Guinea

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the national OBE curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as a alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited of access to secondary and higher educational institutions.

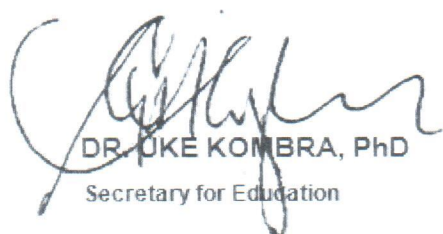
Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



DR. UKE KOMBRA, PhD
Secretary for Education

Unit Introduction



Dear Student,

This is the Fourth Unit of Grade 10 Mathematics Course. It is based on the CDAD Lower Secondary Syllabus and Curriculum Framework for Grade 10.

This Unit consists of four topics:

Topic 1:	Statistical Graphs
Topic 2:	Linear or Straight Line Graphs
Topic 3:	Quadratic Equations and Parabola
Topic 4:	Systems of Equations

In **Topic 1: Statistical graphs**, you will learn to distinguish between discrete and continuous data and present them on a frequency distribution table. You will also learn to identify methods of finding the range and the measures of central tendency. And lastly distinguish between the different types of graphs such as bar and compound graphs, pictograms or pie graphs, and histogram and cumulative frequency graphs.

In **Topic 2: Linear or Straight Line Graphs**, you will learn about linear equations and identify the types of linear graphs. You will also learn to sketch linear graphs using given table of values and equations; find the equations of a line using the form $y = mx + c$ and finally sketch, read and interpret rate and travel graphs.

In **Topic 3: Quadratic Equations and Parabola**, you will revise binomial expressions. You will also learn about the meaning of quadratic equations and identify their properties, solve quadratic equations by completing the squares, by using the quadratic formula. You will also learn to identify the key features of parabola, sketch the different forms of parabola graphically, algebraically and using table of values.

In **Topic 4: System of Equations**, you will learn the meaning of simultaneous system of equations and the solution of a system of equations. You will also learn to find the solution of a simultaneous system of equation by elimination, substitution and graphical method. And finally, sketch or draw the graphs of simultaneous system of equations.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercise are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy using this unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

- Step 1: Start with Topic 1 Lesson 1 and work through it.
- Step 2: When you complete Lesson 1, do Practice Exercise 1.
- Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of TOPIC 1.
- Step 4: Then, revise Lesson 1 and correct your mistakes, if any.
- Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:



Lesson 1: Discrete and Continuous Data

Then go on to the next Lesson. Repeat the process until you complete all of the Lessons in Topic 1.

As you complete each lesson, tick the check-box for that lesson, on the Contents Page, like this ☒. This helps you to check on your progress.

- Step 6: Revise the Topic using Topic 1 Summary, then, do Topic Test 1 in Assignment 4.

Then go on to the next Topic. Repeat the process until you complete all of the four Topics in Unit 4.

Assignment: (Four Topic Tests and a Unit Test)

When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment Book. The Course Book tells you when to do each Topic Test.

When you have completed the four Topic Tests, revise well and do the Topic test. The Assignment Book tells you when to do the Unit Test.

The Topic Tests and the Unit Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

STATISTICAL GRAPHS

Lesson 1:	Discrete and Continuous Data
Lesson 2:	Class Limits and Widths of Continuous Data
Lesson 3:	Range and Measures of Central Tendency
Lesson 4:	Bar and Compound Graphs
Lesson 5:	Pictograms and Pie Graphs
Lesson 6:	Histogram and Cumulative Graphs

TOPIC 1: STATISTICAL GRAPHS

Introduction

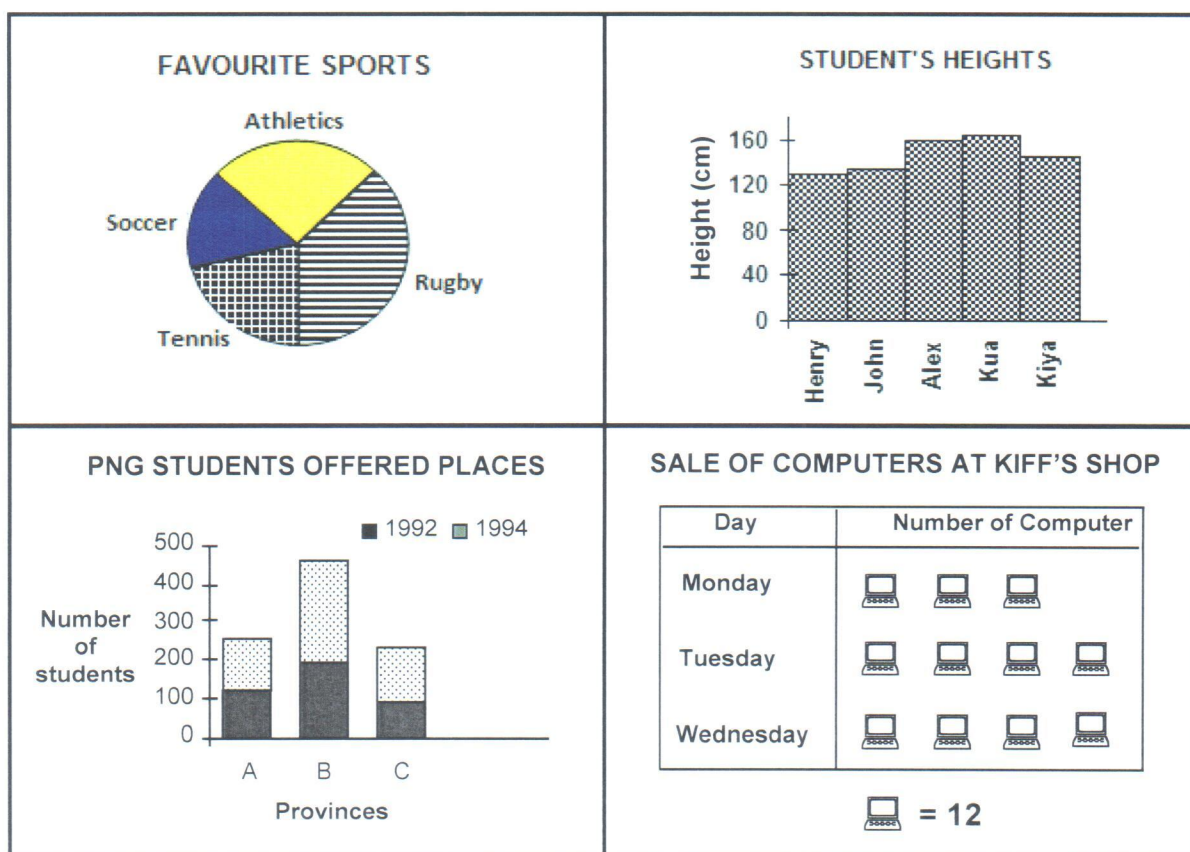


In Grades 7, 8 and 9 Mathematics Courses, you were introduced with the study of graphs and statistics. This Topic will help you revise and then extend your knowledge of the work covered in Formal Mathematics Grade 9 Unit 6 on statistical graphs. Statistics are widely used in many areas to help people make decisions and help them predict the future.

Statistics involves the collection, organization, display and interpretation of numerical information known as **data**.

Generally the data is displayed using tables and graphs.

Graphs are pictures of statistical information. Here are some of them.



In this Topic, you will revise and then extend your knowledge of statistical data and graphs.

Lesson 1: Discrete and Continuous Data



Welcome to the first lesson of your Grade 10 Unit 4.

In this lesson, you will:

- differentiate between Discrete and Continuous Data
 - give examples of Discrete and Continuous Data
 - present Discrete and Continuous data on tables.
-

As you have learnt in the Grade 7, 8 and 9 Mathematics Courses, the study of statistics involves collecting, organizing, displaying and interpreting numerical information.

Raw data is the term used for the numerical information or figures that are initially collected and written down. This data are of two types: Discrete and Continuous.

You were introduced with the meanings of these two types of data in Grade 9.

Let us first have the discrete data.

Discrete Data

In Mathematics, “discrete” means “separate” or “distinct”. Discrete data consists of separate things that can be counted. When we count things, the answers we get are whole numbers. **Whole numbers** are used to describe discrete amounts.

For example, we count the number of cars in a showroom. There may be 50 cars in the showroom, but not $5\frac{3}{4}$ or 21.58 cars, so we cannot use fractions to represent the value of discrete data items.

Discrete data is usually concerned with a limited number of countable values.

Here are other examples of discrete data.

- (a) The number of letters in the alphabet
- (b) The number of students in a class
- (c) The numbers of cars sold in a day
- (d) The number of children in a family
- (e) The number of people absent from work on a workday
- (f) The number of rooms in a house
- (g) The number of books in a room

Now let us look at continuous data.

Continuous Data

Continuous data arises from measurement. Heights, weights, volumes of liquids and temperatures are examples of continuous data.

Continuous data cannot be counted, but it can be measured on some scale and can take any value within that scale. Since, it cannot be measured exactly the value of continuous data item can only be given to a certain **degree of accuracy**.

For example, we could say that the temperature is 35.9°C (correct to 1 decimal place). So the degree of accuracy is only to 1 decimal place, that is, to the nearest tenth of a degree. A person's height could be any value (within the range of human heights), not just certain fixed heights. Time in a race, you could even measure it to fractions of a second.

You can see therefore that **fractions** are often required to describe the value of a continuous data item.

Here are other examples of continuous data.

- (a) The heights of coconut trees
- (b) Weight of rice in a sack
- (c) Depth of rainfall
- (d) Student's ages
- (e) Sailing time from Lae to Port Moresby
- (f) Weights of new born babies
- (g) Length of a leaf

Discrete Data and Frequency Tables

Discrete data is generated by counting; each and every observation is exact. When an observation is repeated, it is counted. The number for which the observation is repeated is called the **frequency** of that observation. The class limits in discrete data are true class limit; there are no class boundaries in discrete data.

If we have only few different items of discrete data, we could normally present or show the data in an ungrouped frequency table, like the one below, which shows the tests results (marked out of 5) of Grade 10 students.

Test Score	Frequency
0	1
1	2
2	3
3	8
4	2
5	2

Example 1

The following are the number of female employees in different commercial banks. Make a frequency distribution.

2, 4, 6, 1, 3, 5, 3, 7, 8, 6, 4, 7, 4, 4, 2, 1, 3, 6, 4, 2, 5, 7, 9, 1, 2, 10, 1, 8, 9, 2, 3, 1, 2, 3, 4, 4, 4, 6, 6, 5, 5, 4, 5, 8, 5, 4, 3, 3, 2, 5, 0, 5, 9, 9, 8, 10, 0, 4, 10, 10, 1, 1, 2, 2, 1, 8, 6, 9, 10

Solution:

The involved variable is “the number of female employees” which is a discrete variable. The largest and smallest values of the given data are 10 and 0 respectively.

Number of Employees (Classes) x	Tally Marks	Branches (Frequency) f
0	II	2
1	III	8
2	IIII	9
3	II	7
4	I	11
5	III	8
6	II	6
7		3
8	I	5
9	I	5
10	I	5

A **frequency table** is a way of summarising a set of data. It is a record of how often each value (or set of values) of the variable in question occurs. It may be enhanced by the addition of percentages that fall into each category.

Sometimes, however, we have too many values to make a row for each one. In that case, we will need to group several values together. When there are many different items of discrete data a **group frequency table** is normally drawn.

Example 2

Suppose 30 students did a Mathematics test (scores marked out of 25).

If we wanted to show the results in an **ungrouped frequency table**, we would need to have in the Test Score column of the table, 31 rows for the scores 0, 1, 2, 3, ..., 24, 25 of the 30 students.

It is much more sensible to group this discrete data into a **grouped frequency table** which will have a much smaller number of rows. In the table on the next page you can see that we have grouped, for example, test scores into groups 0 – 5, 6 – 10, 11 – 15, 16 – 20, 21 – 25, and 26 – 30. Thus we obtain a reasonably sized table, having only six rows of data in it.

SCORES OF 30 STUDENTS IN MATHEMATICS TEST

Test Scores	Frequency
0 - 5	2
6 - 10	3
11 - 15	3
16 - 20	5
21 - 25	10
26 - 30	7

In this table, the scores **between** 0 and 1, 1 and 2, 2 and 3, 3 and 4, 4 and 5 and so on do not exist since discrete data consists on whole numbers.

Continuous Data and Frequency Tables

The grouped frequency table for discrete data shown above is not suitable for continuous data since continuous data is obtained by measurement and therefore values involving fractions and decimals such as 0.5, 2.6, 3.2, 5.8,..., etc. cannot be ignored.

So if we want to present continuous data in a frequency table then it must be shown in a grouped frequency table.

Example 1

Look at the continuous data in the grouped frequency table below, which shows the frequency of the weights of different bags of rice.

We have chosen the first weight group in kilograms to be **0.5 – under 1.5**, that is the “frequency for class 0.5 kg to less than 1.5 kg is 5” or we could write this as “frequency for class 0.5 kg to < 1.5 kg is 5” (since the symbol < means **less than**).

We will write for example, either **“0.5 – under 1.5”** or **“0.5 to < 1.5”** throughout this Unit rather than using the alternative notation 0.5 kg to less than 1.5 kg.

The second weight group is 1.5 – under 2.5, the third weight group is 2.5 – under 3.5, and so on.

WEIGHTS OF DIFFERENT BAGS OF RICE

Weight of bag (kg)	Frequency
0.5 – under 1.5	7
1.5 – under 2.5	5
2.5 – under 3.5	8
3.5 – under 4.5	10

Example 2

A wind blew for 40 days. Its wind speeds, in km/h, were recorded as follows:

15	22	14	12	21	19	34	13	11	0
4	16	23	12	8	18	24	14	17	3
10	12	9	15	20	5	19	13	17	11
16	19	24	7	12	17	14	23	10	14

Draw a grouped frequency distribution table for the speeds using the group “0 to < 5”, 5 to < 10, 10 to < 15 and so on.

Solution:

SPEED OF WIND IN KILOMETRES PER HOUR

Wind Speed	Tally	Frequency
0 to < 5	III	3
5 to < 10	IIII	4
10 to < 15	III III IIII	14
15 to < 20	III III I	11
20 to < 25	III II	7
25 to < 30		0
30 to < 35	I	1
TOTAL		40

NOW DO PRACTICE EXERCISE 1

**Practice Exercise 1**

1. Indicate whether each of the following data is discrete or continuous.

- (a) the time it takes for you to get to school _____
- (b) the number of PNG couples who were married last year _____
- (c) the speed of a bicycle _____
- (d) the number of goals scored by a men's hockey team _____
- (e) your age _____
- (f) the length of time of a telephone call _____
- (g) the annual income of an individual _____
- (h) the number of subjects your school offered last year _____
- (i) the distance between your house and your school _____
- (j) the number of pages in a dictionary _____
- (k) the number of brothers and sisters you have _____
- (l) the number of employees in FODE. _____

2. An age class interval, in years, is 19 – under 25. Which of the people, whose names and ages (to the nearest years) are shown below, belongs to this class interval?

Joshua	19
Jemjem	22
Tenten	25
Daniel	23
Mau	18

Answers: _____, _____, _____

3. Which of the numbers below belongs to the class interval 0.5 – under 1.5?

0.5, 0.9, 4.6, 0.7, 0.2, 1.2, 1.5, 0.4, 1.8, 2.4

Answer: _____

4. A convenience store owner records how many customers enter the store each day over a 25-day period. The results are as follows:

20, 21, 23, 21, 26

24, 20, 24, 25, 22

22, 23, 21, 24, 21

26, 24, 22, 21, 23

25, 22, 21, 24, 21

- (a) Are these discrete or continuous data?

Answer: _____

- (b) Present these data in a frequency distribution table.

- (c) Which result occurs most frequently?

Answer: _____

-
5. Listed below are the weights of Grade 9 students, recorded to nearest kilogram.

47 47 49 49 49 51 51
51 45 45 49 49 45 45
45 51 51 47 51 51 49
47 49 51 51 47 47

Complete the grouped frequency table below using the data above

Weights of Students (kg)	Frequency
43.5 – under 45.5	

6. Fifty staff members of a construction company were surveyed to find out what their fortnightly salary was to the nearest Kina. The results are as follows:

K555 K525 K461 K539 K466 K470 K486 K490 K543 K519
 K481 K502 K528 K459 K548 K521 K517 K463 K478 K443
 K514 K476 K497 K511 K484 K513 K471 K470 K441 K466
 K473 K514 K542 K519 K522 K523 K546 K487 K486 K473
 K527 K470 K440 K564 K499 K523 K484 K463 K461 K437

- (a) Are these data discrete or continuous?

Answer: _____

- (b) Tabulate these data in the grouped frequency distribution table shown below, the classes are: 420 - < 440, 440 - < 460, etc. for these data.

Fortnightly Salary	Tally Marks	Frequency (f)
420 - < 440		
440 - < 460		
Total		

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Class Limits and Widths for Continuous Data



You have learned to identify discrete and continuous data in the last lesson. You have also learned to organize discrete and continuous data into the frequency distribution tables.



In this lesson, you will:

- define class limits and class widths for continuous data
- group continuous data into a number of classes by determining class width
- draw a grouped frequency tables.

As you learned in the previous lesson, discrete data **can** be counted and continuous data **must** be in a frequency table, presented in a grouped frequency distribution table.

Now, let us consider an example where continuous data is involved.

In a class of 40, every student was asked to bring a flower to school and then measured the length of the flowers (to the nearest mm) and record the length in centimetres (cm).

The results are listed in the raw data below:

6.0	7.2	5.9	5.8	5.4	7.9	4.6	5.7	6.2	3.2
5.2	9.2	5.7	6.3	7.2	3.7	6.8	6.3	5.2	4.2
4.4	5.3	8.9	5.0	3.8	4.6	7.7	5.4	3.8	7.5
8.7	5.6	4.9	6.5	4.6	8.4	6.3	3.4	4.7	4.6

To summarize this data, you need to draw a grouped frequency distribution table showing the results into different intervals of numbers called **classes**. Each group or class contains more than one score value called the **class interval**. This class interval contains the number of score values.

Here are the steps for constructing the grouped frequency distribution table.

1. Compute the range. $\text{Range} = \text{highest value} - \text{lowest value}.$

You can see from the data that the lengths of the flowers vary from 3.2 cm to 9.2 cm, or using whole appropriate numbers, from lengths equal or greater than 3 cm to less than 10 cm (using whole numbers makes calculations easier!).

Now using 3 as the lowest value and 10 as the highest value, we can compute the range as

$$\begin{aligned}
 \text{Range} &= \text{highest value} - \text{lowest value.} \\
 &= 10 \text{ cm} - 3 \text{ cm} \\
 &= 7 \text{ cm}
 \end{aligned}$$

- Decide on the number of classes your frequency table will contain. Usually 5 and 20 is good.

In the given example, since the range is **7** cm, it is sensible to group the data into **7** classes so that we don't have to deal with fractions.

- Determine the class width by dividing the range by the number of classes. Round your result up to a convenient value.

$$\begin{aligned}\text{Hence, Class width} &= \frac{\text{Range}}{\text{number of classes}} \\ &= \frac{7}{7} \\ &= 1 \text{ cm}\end{aligned}$$

- Locate the **starting point** by selecting either the lowest value or a convenient value slightly less than the lowest value as the lower limit of the first class.
- Add the class width to the starting point to get the second lower class limit. And so on.

Therefore, the class intervals we will use are: 3 cm – under 4 cm, 4 cm – under 5 cm, 5 cm – under 6 cm and so on until you reach the class containing the highest value.

- List the lower class limits in vertical column, along with the upper class limits.
- Complete the table by counting up class frequencies and filling them in.

The results are then sorted into these class intervals giving us the grouped frequency distribution table as shown below..

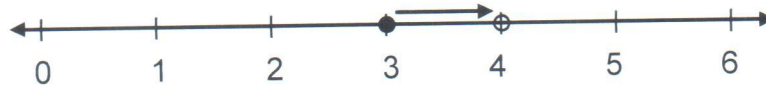
Length (cm)	Tally Marks	Frequency (f)
3 – under 4		5
4 – under 5	– III	8
5 – under 6	– – I	11
6 – under 7	– II	7
7 – under 8		5
8 – under 9		3
9 – under 10	I	1
Total		40

Class Limits

The boundaries between one class and the next are called **class limits** or sometimes called **class boundaries**. Each class has an **upper class limit** and a **lower class limit**.

Look again at the grouped frequency table on the previous page, the first class is **3 – under 4**. The Lower class limit is **3**, i.e. the number on the left of the class '3 – under 4'. And look at the Upper class limit, which is the number on the right of the class '3 – under 4', that is: the upper limit below **4**.

The Upper Limit below would be any of these; 3.96, 3.999, or 3.9955 or any other number close to but less than 4 as shown on the number line below.



We obviously need to round the number, in this particular case to the next whole number, i. e. 4. So we take the Upper class limit when rounded to be equal to 4.

In the same way, for the class interval '**4 – under 5**':

Lower class limit = **4**, Upper class limit = **5** (rounded)

Similarly, for the class interval '**5 – under 6**':

Lower class limit = **5**, Upper class limit = **6** (rounded)

Let us again look the first column (Length) at the grouped frequency table we draw on the previous page.

Length (cm)	
3 – under	4
4 – under	5
5 – under	6
6 – under	7
7 – under	8
8 – under	9
9 – under	10

Lower Class Limits or Boundaries

Upper Class Limit or Boundaries

- Lower Class Limit or boundary is the number on the left of each individual class interval.
- Upper Class Limit or boundary is the number on the right of each individual class interval.

Class width

To determine the class width, divide the range by the number of classes. The range is the difference between the highest value and lowest value. Hence,

$$\text{Class width} = \frac{\text{highest value} - \text{lowest value}}{\text{number of classes}}$$

For our given data,

$$\begin{aligned}\text{Class width} &= \frac{10 - 3}{7} \\ &= 1\end{aligned}$$

The class width is also the difference between the lower limits of two consecutive classes or the upper limits of two consecutive classes. It is not the difference between the upper and lower limits of the same class.

Class width is the difference between two consecutive lower limits or two consecutive upper limits.

Look again at the grouped frequency table we drew on the previous page.

Length (cm)	Tally Marks	Frequency (f)
3 – under 4		5
4 – under 5	– III	8
5 – under 6	– – I	11
6 – under 7	– II	7
7 – under 8		5
8 – under 9		3
9 – under 10	I	1
Total		40

The first class in the above table is: '3 – under 4', where 3 is the Lower class limit and 4 is the Upper class limit.

The second class in the above table is: '4 – under 5', where 4 is the lower class limit and 5 is the upper class limit.

3 and 4 are two consecutive lower class limits and 4 and 5 are two consecutive upper class limit.

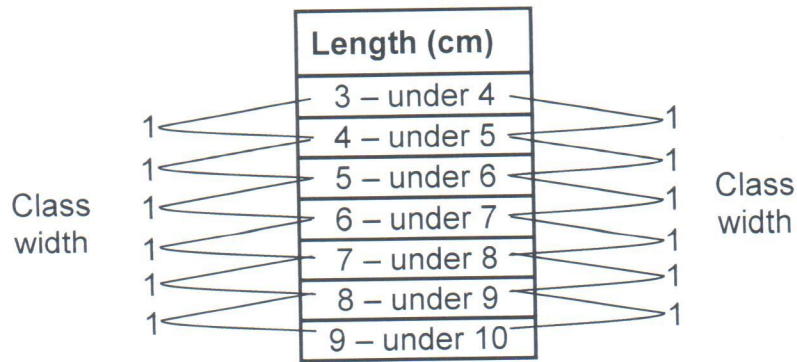
Therefore: Class width = difference between two consecutive lower class limits **OR** two consecutive upper class limit.

$$= 4 - 3 \text{ OR } 5 - 4$$

So, Class width = **1 cm**

Similarly, the Class widths that follow will also be 1 cm. Thus the Class width is the same for each of the classes.

See illustration of these on the next page.



The fact that class widths are equal it helps us to find out the class limits and class intervals for any frequency table for continuous data.

Here are some examples.

Example

Complete the table shown below.

Class (kg)	Frequency
9.6 – under 14.6	21
14.6 – under 19.6	17
	15
	4
	3

To complete the table, we first find the class width.

Class width = difference between two consecutive lower class limits **OR** two consecutive upper class limit.

$$= 14.6 - 9.6 \text{ OR } 19.6 - 14.6$$

$$= 5 \text{ kg}$$

The third class interval is from 19.6 – under $(19.6 + 5)$ i.e. from 19.6 – under 24.6 kg

The fourth class interval is from 24.6 – under $(24.6 + 5)$ i.e. from 24.6 – under 29.6 kg

The completed frequency table is shown below.

Class (kg)	Frequency
9.6 – under 14.6	21
14.6 – under 19.6	17
19.6 – under 24.6	15
24.6 – under 29.6	4
29.6 – under 34.6	3

Example 2

Below is the list of times of phone calls, measured to the nearest tenth of a minute.

3.8 12.4 13.7 1.5 15.6 8.5 26.3
6.9 16.2 16.5 17.9 10.5 21.8

Time is continuous. Draw a grouped frequency table for this data, using the class limits 0 – under 10 minutes, and so on.

Time (minutes)	Frequency
0 – under 10	4
10 – under 20	7
20 – under 30	2

Example 3

Below represents the distances covered by a group of 40 cyclists in a sponsored bicycle ride.

7 10 10 12 11 9 11 9 11 13
8 12 9 13 6 8 10 11 12 12
10 15 11 9 14 9 12 11 8 10
14 13 11 10 7 8 9 10 10 10

- (a) Draw a grouped frequency table with class intervals 5 – under 7 km, 7 – under 9 km, etc. for the above data.

Distance (km)	Frequency
5 – under 7	1
7 – under 9	6
9 – under 11	15
11 – under 13	12
13 – under 15	5
15 – under 17	1

- (b) What is the Class width?

$$\begin{aligned}\text{Class width} &= 7 - 5 \\ &= 2\end{aligned}$$

NOW DO PRACTICE EXERCISE 2



Practice Exercise 2

1. A fisheries student working on a project measured the length in centimeters of 40 fish. The results are shown below:

13.6	14.1	14.0	15.8	16.2	16.3	15.4	13.6	17.5	16.2
14.2	14.8	15.6	16.3	16.6	14.3	16.5	16.3	14.9	15.2
16.1	15.8	16.2	16.5	14.2	14.3	15.8	15.7	14.7	14.6
14.7	13.9	15.3	14.6	15.7	15.7	15.7	13.9	16.0	16.1

The student then wanted to draw a grouped frequency table of the above data with **5** as the **number of class intervals** using **13** as the **lowest value** and **18** as the **highest value**.

- (a) What is the range?

Answer: _____

- (b) What is the class width?

Answer: _____

- (c) What would the first class interval be?

Answer: _____

- (d) From the above data, complete the grouped frequency table shown below.

Length (km)	Frequency
13– under 14	
Total	40

2. Given below is a list of the distances, measured to the nearest tenth of a kilometer, covered by a group of 20 students on one hour at a school sports carnival.

1.8	3.2	4.7	2.3	1.4
5.9	3.1	2.9	1	6.5
2	3.9	2.8	4.6	4.9
2.6	1.5	5.8	3.7	1.9

Using the above data, complete the grouped frequency table below

Distance (km)	Frequency
1 – under 3	
3 – under 5	
5 – under 7	
Total	

3. Plant scientists developed different varieties of corns that have a rich content of lysine which is a nutritious animal feed. A group of chicks were given this food to test the quality. Weight gains (in grams) of these chicks after 21 days are as recorded:

380.3	321.2	366.0	356.3	349.0
337.1	399.2	384.5	410.4	329.6
350.0	340.3	324.5	396.8	412.7
420.9	382.4	318.0	344.5	438.0

- (a) Find the range.
- (b) Using 7 as the number of desired class intervals. Find the class width.
- (c) Construct the class limits (weight gains) with the class width, so that the lowest and the highest values are included on a frequency distribution table.

Height (in cm)	Frequency
318 -	
Total	20

- (d) Find the class interval in which the weight increase is the highest.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.
--

Lesson 3: Range and Measures of Central Tendency



You have learned to find the class limits and class width for continuous data in the previous lesson.



In this lesson, you will:

- identify the method of finding the mean of continuous data
- find an estimate of the mean using class centers of a set of continuous data presented on a frequency distribution table
- find the median class, modal class and class range of a set of continuous data.

Basically, the method of estimating the mean for a set of continuous grouped data is the same when finding the estimated mean for a set of discrete grouped data as you did in FODE Grade 9 Unit Topic 3, Lesson 14.

Let us learn to find the mean of a set of continuous data in the following set of data shown in the frequency table below.

The table shows the result of a survey of the number of hours of watching television by a group of 30 people in one week.

**NUMBER OF HOURS OF 30 PEOPLE
WATCHING TELEVISION IN A WEEK**

Number of hours	Frequency
0 – under 5	6
5 – under 10	9
10 – under 15	7
15 – under 20	5
20 – under 25	2
25 – under 30	1
N = 30	

Looking at the table above, we can find an estimate of the mean number of hours watching. But we cannot find the exact value of the mean number of hours watching itself because we are not given each individual's viewing time.

For example, in the table we can see that there are 6 people who watch 0 – under 5 hours of television but we do not know the actual number of hours each person watched in this interval of time.

To estimate the mean number of hours watched by the 30 people, we first need to find or calculate the midpoint value (Class center) of each class, using the steps and formula on the next page.

Steps:

- (a) add the lower class boundary and the upper class boundary in the class
- (b) divide the result by 2.

$$\text{Class center (Midpoint)} = \frac{\text{lower class boundary} + \text{upper class boundary}}{2}$$

Formula:

$$M = \frac{LS + HS}{2}$$

So for the grouped frequency table on the previous page we have the:

$$\text{first class:} \quad \text{Midpoint} = \frac{0 + 5}{2} = 2.5$$

$$\text{second class:} \quad \text{Midpoint} = \frac{5 + 10}{2} = 7.5$$

$$\text{third class:} \quad \text{Midpoint} = \frac{10 + 15}{2} = 12.5$$

$$\text{fourth class:} \quad \text{Midpoint} = \frac{15 + 20}{2} = 17.5$$

$$\text{fifth class:} \quad \text{Midpoint} = \frac{20 + 25}{2} = 22.5$$

$$\text{sixth class:} \quad \text{Midpoint} = \frac{25 + 30}{2} = 27.5$$

Now, having all the class centers or midpoints we extend the previous table to include a column for the following:

- (1) class center or midpoint (M) column
- (2) the product of frequency(f) and midpoint column(fM)
- (3) row totals for frequency (N) and
- (4) the cumulative frequency
- (5) the sum of the products of frequency and midpoint ($\sum fM$).

Number of hours	Frequency (f)	Cumulative Frequency	Class-center(M)	fM
0 – under 5	6	30	2.5	5 x 2.5 = 12.5
5 – under 10	9	24	7.5	9 x 7.5 = 67.5
10 – under 15	7	15	12.5	7 x 12.5 = 87.5
15 – under 20	5	8	17.5	6 x 17.5 = 105
20 – under 25	2	3	22.5	2 x 22.5 = 45
25 – under 30	1	1	27.5	1 x 27.5 = 27.5
	N= 30			$\sum fM = 345$

Now we can find an estimate of the mean of this continuous data using the formula for the mean of a set of grouped data you probably remember from Grade 9 Unit 3 Topic 3 (but replacing the word “mean” by “estimate of mean”).

$$\text{Estimate of mean} = \frac{\text{sum of } fM \text{ products}}{\text{sum of frequencies}}$$

$$\bar{X} = \frac{\sum fM}{N}$$

Where: f = is the frequency of each class
 M = is the class center or midpoint of each class
 N = is total number of frequency

Thus using the values in the previous table, we have:

$$\bar{X} = \frac{\sum fM}{N}$$

$$\bar{X} = \frac{345}{30}$$

$$\bar{X} = 11.5 \text{ hours}$$

Therefore, the estimated mean number of hours viewing is 11.5 hours.

MEDIAN

Now we will look at estimating the median of a set of continuous grouped data from a frequency table.

From a grouped frequency table of a set of continuous data we can only find the **median class**, the class in which the median lies; not the exact value of the median, since we do not know the value of the individual items in a class.

The median class contains either:

- (i) the middle score for an odd number of scores
- or
- (ii) the mean of the two middle scores for an even number of scores.

In the above example there are 30 scores, so, since there are even number of scores and the median class contains the mean of the two middle scores, the 15th score

($\frac{30}{2} = 15$) and the 16th score.

The 14th score (5 + 9) lies at the end of the class 5 – under 10 hours.

The 21st score (5 + 9 + 7) lies at the end of the class 10 – under 15 hours.

Hence both the 15th and 16th scores lie in the class 10 – under 15 hours, so the mean of the 15th and 16th scores must lie in the class 10 – under 15 hours.

So, the median class is 10 – under 15 hours.

But if we need to estimate a single median value we can use these formulas:

(a)

$$\text{Estimated Median} = L + \frac{\left(\frac{N}{2}\right) - F_b}{f_m} (i) \quad \leftarrow \text{Median from below}$$

Where:

- **L** is the lower class boundary of the median class
- **N** is the total number of data
- **F_b** is the cumulative frequency below the median class
- **f_m** is the frequency of the median class
- **i** class width or class size

(b)

$$\text{Estimated Median} = U - \frac{\left(\frac{N}{2}\right) - F_a}{f_m} (i) \quad \leftarrow \text{Median from above}$$

Where:

- **U** is the upper class boundary of the median class
- **N** is the total number of data
- **F_a** is the cumulative frequency above the median class
- **f_m** is the frequency of the median class
- **i** class width or class size

Thus, using the values in the table on page 26:

For (a)

- **L** = 10
- **N** = 30
- **F_b** = 15
- **f_m** = 7
- **i** = 5

$$\begin{aligned} \text{Estimated median} &= 10 + \frac{15 - 15}{7} \times 5 \\ &= 10 + \frac{0}{7} \times 5 \\ &= 10 \text{ hours} \end{aligned}$$

For (b)

- **U** = 15
- **N** = 30
- **F_a** = 8
- **f_m** = 7
- **i** = 5

$$\begin{aligned} \text{Estimated median} &= 15 - \frac{15 - 8}{7} \times 5 \\ &= 15 - \frac{7}{7} \times 5 \\ &= 10 \text{ hours} \end{aligned}$$

Therefore, the median is 10 hours.

MODE

We can easily identify the modal group (the group with the highest frequency).

In our example, the group with the highest frequency is the class **5 – under 10** hours.

We can say “the modal class is 5 – under 10 hours”.

But the actual may not even be in that group! Or there may be more than one mode. Without the raw data we don't really know.

But we can estimate the Mode using the formula:

$$\text{Estimated Mode} = L + \frac{F_1 - F_0}{2F_1 - F_1 - F_2} (i)$$

where:

- L is the lower class boundary of the modal class
- F_1 is the frequency of the modal class
- F_2 is the frequency after the modal class
- F_0 is the frequency below the modal class
- i class width or class size

Using the values in our example:

- $L = 5$
- $F_1 = 9$
- $F_2 = 7$
- $F_0 = 6$
- $i = 5$

$$\begin{aligned} \text{Estimated mode} &= 5 + \frac{9 - 6}{2(9) - 9 - 7} \times 5 \\ &= 5 + \frac{3}{18 - 16} \times 5 \\ &= 5 + \frac{15}{2} \\ &= 5 + 7.5 \\ &= 12.5 \text{ hours} \end{aligned}$$

CLASS RANGE

You will remember that the range gives us an idea of the spread of the data.

As with the estimated mean, median class and modal class, since we do not know the actual values of the data items in a grouped frequency table we cannot find the actual value of the range. We can only find the **class range**, which is the difference between the upper limit of the top class and the lower limit of the bottom class.

That is:

$$\text{Class range} = \text{Upper limit of top class} - \text{Lower limit of bottom class}$$

Thus to find the class range of the table in page 26:

Upper Limit of the top class = 30

Lower limit of the bottom class = 0

Therefore, the Class range of the above example = $30 - 0 = 30$ hours.

As mentioned in the beginning of this lesson, Continuous data can be treated in exactly the same way as discrete data, but with one **important difference**. The difference concerns the **class boundaries**.

Example:

Jessie harvested fifty eggplants in his garden. He measured their lengths (to the nearest mm) and grouped the results as shown in the frequency table below.

Length (mm)	Frequency (f)
150 - 154	5
155 - 159	2
160 - 164	6
165 - 169	8
170 - 174	9
175 - 179	11
180 - 184	6
185 - 189	3

Now, let us find out what does “155 – 159” mean?

We have the clue which is “**to the nearest mm**”.

- A length of 154.5 mm would be rounded up to 155 mm (and placed in 155 – 159).
- Similarly 159.49 mm would be rounded down to 159 mm (and be placed in 155 – 159).

So, lengths from 154.5 up to **(but not including)** 159.5 get placed in 155 - 159

And so there are two types of numbers at the beginning and end of the continuous data “155 – 159”:

- (i) the lower class boundary of 155 and the upper boundary of 159
- (ii) the lower class limit of 154.5 and upper class limit of 159.5

Also note that the upper class limit of one class interval is the lower class limit of the next class interval.

So, how does this affect our calculations for the mean, median and mode?

- The mean is not affected
- But the median and the Mode now have **lower class limit (L)** rather than the lower class boundary.

Now let us find the estimated mean.

First, we have to find the **class centres or midpoints** of each class intervals. As we learnt, the midpoint is computed by the formula:

$$M = \frac{LS + HS}{2}$$

So, for the first class interval of the above table:

$$\begin{aligned} M &= \frac{LS + HS}{2} \\ M &= \frac{150 + 154}{2} \\ M &= \frac{304}{2} \\ M &= 152 \text{ mm} \end{aligned}$$

The third column of the table below shows all the midpoints of each class intervals.

Length (mm)	Frequency (f)	Midpoints M
150 - 154	5	152
155 - 159	2	157
160 - 164	6	162
165 - 169	8	167
170 - 174	9	172
175 - 179	11	177
180 - 184	6	182
185 - 189	3	187
	N= 50	

Now, we get the product of each midpoint and the corresponding frequency within its interval (fM).

So for the first interval 150 – 154, we have: $fM_1 = 5 \times 152 = 760$

For the interval 155 – 159 we have $fM_2 = 2 \times 157 = 314$; the third, $fM_3 = 6 \times 162 = 972$ the fourth, $fM_4 = 8 \times 167 = 1336$; and so on.

Length (mm)	Frequency (f)	Midpoints M	fM
150 - 154	5	152	760
155 - 159	2	157	314
160 - 164	6	162	972
165 - 169	8	167	1336
170 - 174	9	172	1548
175 - 179	11	177	1947
180 - 184	6	182	1092
185 - 189	3	187	561
	N= 50		$\Sigma fM = 8530$

Now, we can compute the estimated mean, median and mode.

1. The Mean.

$$\text{Estimated mean } (\bar{X}) = \frac{\sum fM}{N}$$

$$\bar{X} = \frac{8530}{50}$$

$$\bar{X} = 170.6 \text{ mm}$$

2. The Median

From the table, the **median class** contains the mean of the 25th and the 26th length, so is in the interval class group **170 – 174**.

To calculate the estimated single median value, we do the following steps:

- Determine the class limits

For the group 150 – 154, the lower limit is 149.5 and the upper limit is 154.5
For the group 155 – 159, the lower limit is 154.5 and the upper limit is 159.5
And so on.

The fifth column on the table below shows the class limits.

Length (mm)	Frequency (f)	Midpoints M	fM	Class Limits	
				Lower	Upper
150 - 154	5	152	760	149.5	154.5
155 - 159	2	157	314	154.5	159.5
160 - 164	6	162	972	159.5	164.5
165 - 169	8	167	1336	164.5	169.5
170 - 174	9	172	1548	169.5	174.5
175 - 179	11	177	1947	174.5	179.5
180 - 184	6	182	1092	179.5	184.5
185 - 189	3	187	561	184.5	189.5
	N = 50		$\sum fM = 8530$		

- Calculate the estimated single median using the formula:

(a)

$$\text{Estimated Median} = L + \frac{\left(\frac{N}{2}\right) - F_b}{f_m} \quad (i)$$

where:

- **L** is the lower class limit of the median class
- **N** is the total number of data
- **F_b** is the cumulative frequency below the median class
- **f_m** is the frequency of the median class
- **i** class width or class size

From the table: $L = 169.5$; $N = 50$; $Fb = (5+2+6+8) = 21$; $f_m = 9$; $i = 5$
 Substituting these values in the formula:

$$\begin{aligned}\text{Estimated median} &= 169.5 + \frac{\frac{50}{2} - 21}{9} \times 5 \\ &= 169.5 + \frac{25 - 21}{9} \times 5 \\ &= 169.5 + 2.22 \\ &= \mathbf{171.7 \text{ mm}} \text{ (to 1 decimal)}\end{aligned}$$

(b)

$\text{Estimated Median} = U - \frac{\left(\frac{N}{2}\right) - F_a}{f_m} \text{ (i)}$
--

where:

- U is the upper class limit of the median class
- N is the total number of data
- F_a is the cumulative frequency above the median class
- f_m is the frequency of the median class
- i class width or class size

From the table: $U = 174.5$; $N = 50$; $F_a = (11+6+3) = 20$; $f_m = 9$; $i = 5$
 Substituting these values in the formula:

$$\begin{aligned}\text{Estimated median} &= 174.5 - \frac{\frac{50}{2} - 20}{9} \times 5 \\ &= 174.5 - \frac{25 - 20}{9} \times 5 \\ &= 174.5 - 2.77 \\ &= \mathbf{171.7 \text{ mm}} \text{ (to 1 decimal)}\end{aligned}$$

Therefore, the estimated single median value is **171.7 mm**.

3. Mode

From the table, the **modal class** is the class with the highest frequency, which is the **175 – 179**:

But we can estimate the estimated Mode using the formula:

$\text{Estimated Mode} = L + \frac{F_1 - F_0}{2F_1 - F_1 - F_2} \text{ (i)}$
--

where:

- L is the lower class limit of the modal class
- F_1 is the frequency of the modal class
- F_2 is the frequency after the modal class
- F_0 is the frequency below the modal class
- i class width or class size

From the table, $L = 174.5$; $F_1 = 11$; $F_2 = 6$; $F_0 = 9$; $i = 5$

Hence, substituting these values to the formula, we have

$$\begin{aligned}\text{Estimated mode} &= 174.5 + \frac{11 - 9}{2(11) - 9 - 6} \times 5 \\ &= 174.5 + \frac{2}{22 - 15} \times 5 \\ &= 174.5 + \frac{10}{7} \\ &= 174.5 + 1.42 \\ &= \mathbf{175.9 \text{ mm}} \text{ (to 1 decimal)}\end{aligned}$$

Therefore, the estimated mode is 175.9 mm.

4. Class range

Class range = Upper limit of top class – Lower limit of bottom class

Thus to find the class range of the above table:

Upper Limit of the top class = 189.5

Lower limit of the bottom class = 149.5

Therefore, the class range = $189.5 - 149.5 = 40 \text{ mm}$

Remember:

For grouped data, we cannot find the exact Mean, Median and Mode, we can only give estimates.

To estimate the mean, use the midpoints of the class intervals.

For continuous data, to estimate the median and mode, use the class limits rather than boundaries.

NOW DO PRACTICE EXERCISE 3



Practice Exercise 3

1. What would be the class center or midpoints of these class intervals?

a. 11 – under 15

Answer: _____

b. 13 – under 18

Answer: _____

c. 8 – under 32

Answer: _____

2. The following frequency table shows the ages of the number of tourists who visited Papua New Guinea in 2010.

Age Group	Frequency
1 – under 11	400
11 – under 21	800
21 – under 31	2600
31 – under 41	4600
41 – under 51	4000
51 – under 61	2200
61 – under 71	1400

(a) Extend the above frequency table by filling in the columns.

Age Group	Frequency (f)	Class centers (M)	fM
1 – under 11	400		
11 – under 21	800		
21 – under 31	2600		
31 – under 41	4600		
41 – under 51	4000		
51 – under 61	2200		
61 – under 71	1400		
N=			$\sum fM =$

(b) Find the estimated mean age (to 1 decimal place) of the people who visited PNG in 2010

Answer: _____

(c) Which is the Median class?

Answer: _____

(d) Which is the Modal class?

Answer: _____

(e) What is the Class range?

Answer: _____

(f) What percentage of the people who visited PNG in 2014 were aged 31 or below?
(Write your answer to the nearest %).

Answer: _____

3. A survey of the distance travelled (to the nearest km) to university by a group of students are shown in the frequency distribution table below.

(a) Complete the table.

Distance in Km	Frequency (f)	Midpoints (M)	Frequency x Midpoints (fM)
1 – under 4	18	2.5	45
4 – under 7	22		
7 – under 10	16		
10 – under 13	15		
13 – under 16	8		
16 – under 19	12		
19 – under 22	6		
22 – under 25	3		
	N =		$\sum fM =$

(b) Using the data in your table find the:

(i) Modal class

Answer: _____

(ii) Mean distance travelled

Answer: _____

(iii) Percentage of students who travel less than 13 km to the university.

Answer: _____

4. The heights (in cm) of a group of thirty students were measured and recorded.

Height (cm)	Frequency (f)
140 – 144	2
145 – 149	4
150 – 154	3
155 – 159	4
160 – 164	5
165 – 169	5
170 – 174	3
175 – 179	4

- (a) Complete the frequency distribution table by supplying the data of the empty columns.

Height (cm)	Frequency (f)	Midpoints (M)	f x M (fM)	Class limits	
				Lower	Upper
140 – 144	2				
145 – 149	4				
155 – 159	4				
160 – 164	5				
150 – 154	3				
165 – 169	5				
170 – 174	3				
175 – 179	4				
N =			$\Sigma fM =$		

- (b) Using the data in your table, find the following:

(i) Class Size

Answer: _____

(ii) Upper class limit of the top class

Answer: _____

(iii) Lower class limit of the bottom class

Answer: _____

(iv) Class range

Answer: _____

(v) Whether or not the modal class exists (answer YES or NO).

Answer: _____

(vi) Estimated mean height of the students (to 1 decimal place)

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 4: Bar and Compound Graphs



You have learned about the range and the measures of central tendency in the previous lesson.



In this lesson, you will:

- differentiate between bar graphs and compound graphs
- draw, read and interpret bar graphs and compound graphs

You are aware that graphs are really pictures of statistical information and that there are several graphs that represent data, for example, by using bar graphs and compound graphs.

We will look at bar graphs and compound graphs in this lesson

Bar Graphs

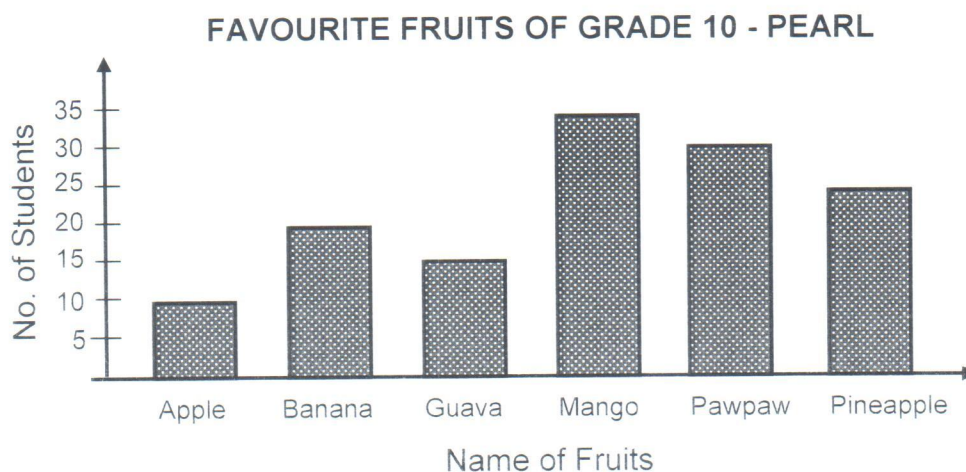
You know already from what you have previously studied in your mathematical course that a Bar Graph is a graph that uses separate bars to show statistical information.

There are two types of bar graph:

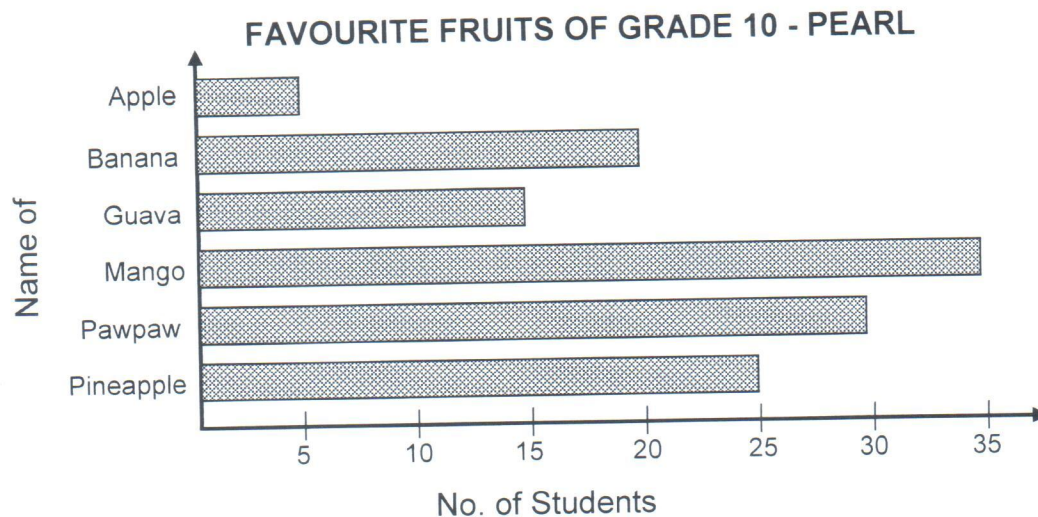
1. Column graphs (vertical bars) and
2. Horizontal bar graphs.

Below are examples of these two types of bar graph each showing the same data, the favorite fruits of Grade 10 – Pearl students.

Column Graph or Vertical Bar Graph



Horizontal Bar Graph



Whether we use a column graph or a horizontal bar graph to show the same data normally depends on how the graph fits on a page.

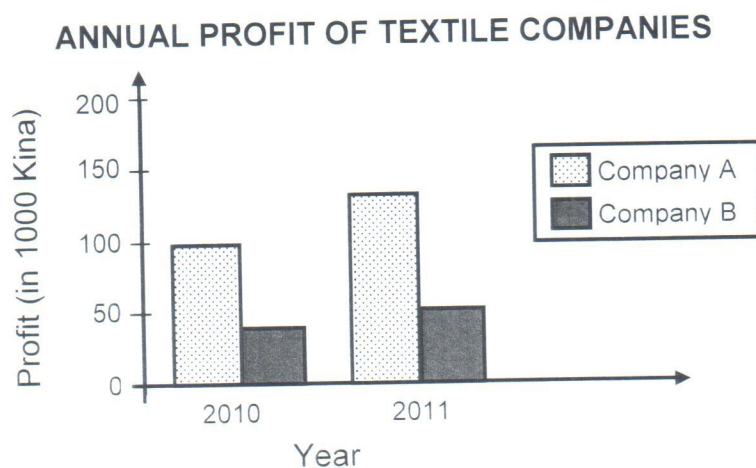
In the graphs above, we have been able to use a larger scale (because we can make use of the whole width of the page) for the number of students on the horizontal bar graph compared to the scale for the vertical graph. Consequently the horizontal bar graph is a better choice in the above example as the larger scale makes the information displayed clearer.

Compound Graphs

You learnt in Grade 8 and Grade 9, that a compound graph is useful in comparing two or more different sets of data. In compound graphs, we can either draw different bars beside each other. Or we can draw different bars on top of each other.

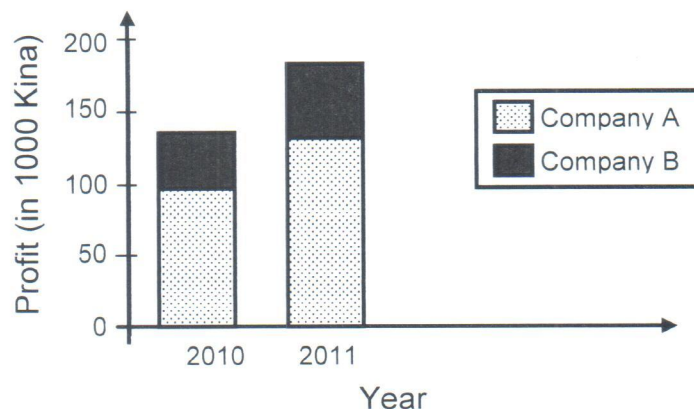
Here is an example of the same data (annual profit of a textile company) presented in these two ways.

- (i) Compound graph: different bars beside each other.



- (ii) Compound graph: different bars stacked one on top of another.

ANNUAL PROFIT OF TEXTILE COMPANIES



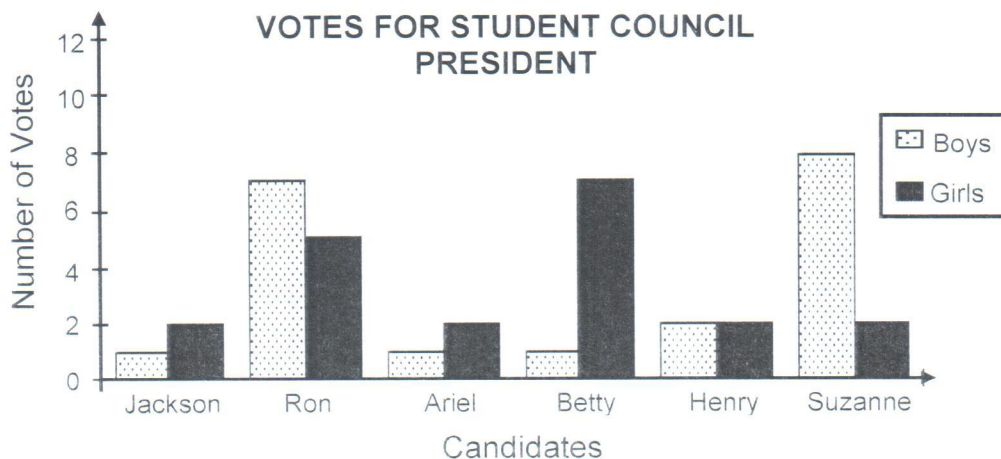
The above graphs show that it is useful to use the graph by drawing bars beside each other when you want to compare the profit of one company with another company. Alternatively, if you wanted to find the total profit for one year, then the graph in which the bars are on top of each other would be more helpful.

Here is a second example. The table below shows the votes received by six candidates for the position of School Student Council (SSC) President.

Candidate	Girls	Boys	Total
Jackson	1	2	3
Ron	7	5	12
Ariel	1	2	3
Betty	1	7	8
Henry	2	2	4
Suzanne	8	2	10

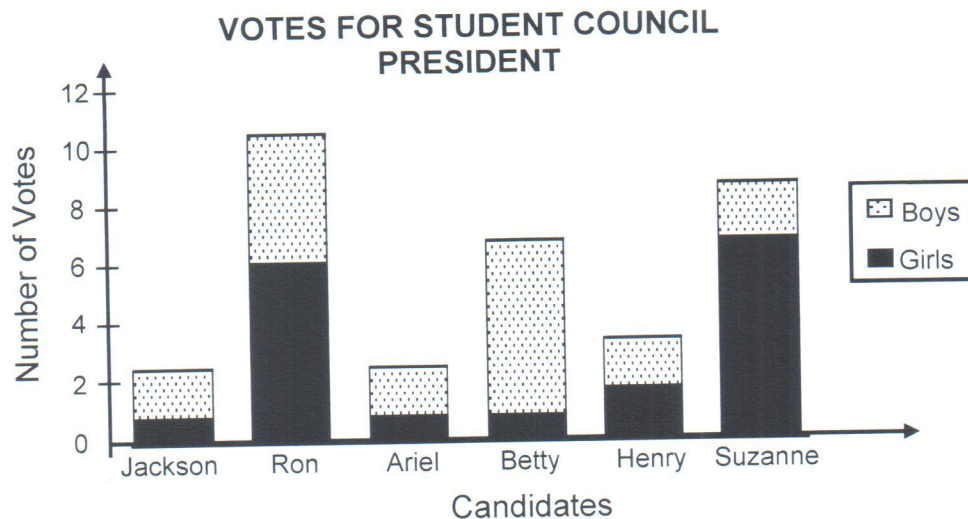
Depending on what information is required from the above data we can represent it in a compound graph in two different ways.

To find out the most popular candidate with the girls and the most popular candidate with the boys, it is better for comparison of the data to draw a compound graph with the different bars beside each other:



You can see clearly from the graph on the previous page that the most popular candidate with the girls was Betty and the most popular candidate with the boys was Suzanne.

On the other hand, to find out, who became the School Student Council (SSC) President, we need to know the total votes for each candidate. So in this case it is best to draw a graph in which the bars are on top of each other (the stacked graph) since the graph will show clearly that Ron became the School Student Council (SSC) President.



Now study the following examples.

Example 1

- For a diving competition there were five judges. The judges A, B, C, D, E, gave points out of 10 for four competitors: Sam, Tom, Kline and Claire. The results are shown in the table below.

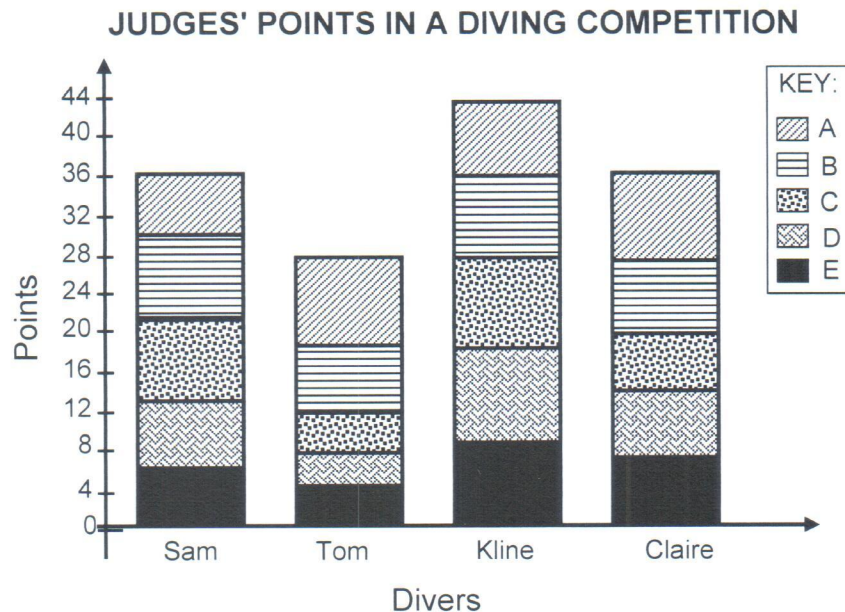
Divers	Judges				
	A	B	C	D	E
Sam	6	7	9	7	8
Tom	4	5	3	8	7
Kline	9	10	8	8	9
Claire	7	7	8	6	8

Questions:

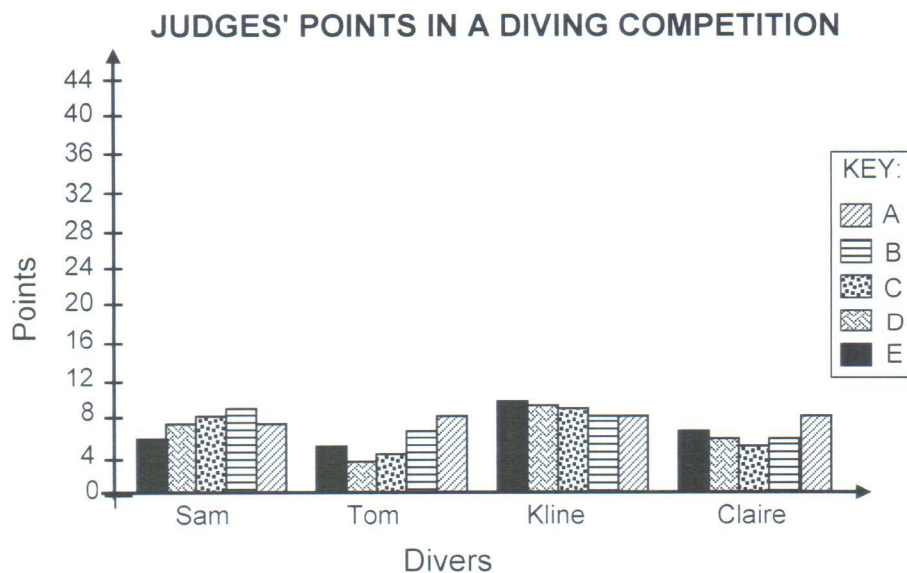
- Draw a compound graph (stacked) to illustrate the data shown on the table.
- Using the same as the graph you drew in (a), draw a compound graph (columns side by side) to illustrate the data.
- What do you think about the scales used in the two graphs?
- Re-draw the graph in (b) using a better scale.

Let us do the working out.

- (a) Draw a compound graph (stacked) to illustrate the data.



- (b) Using the same scale as the graph you drew in part (a) draw a compound graph (columns side by side) to illustrate the data.

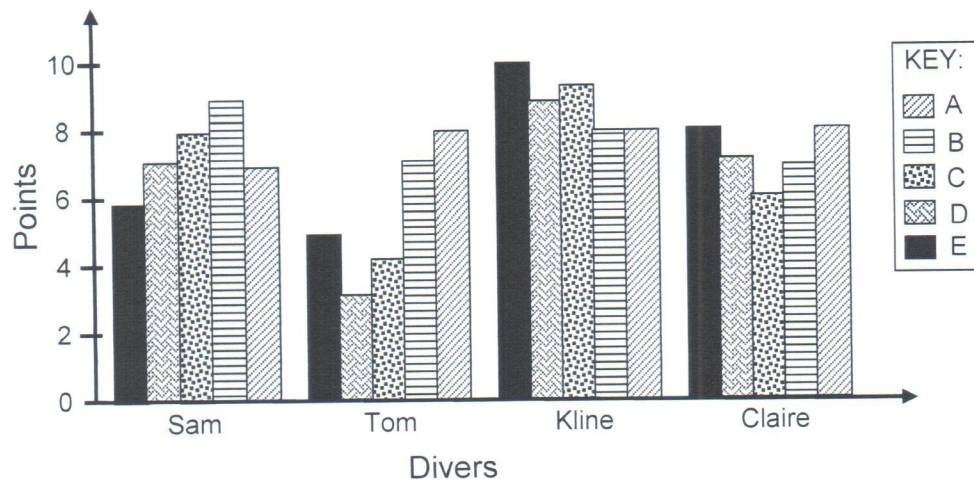


- (c) What do you think about the scales used in the two graphs above?

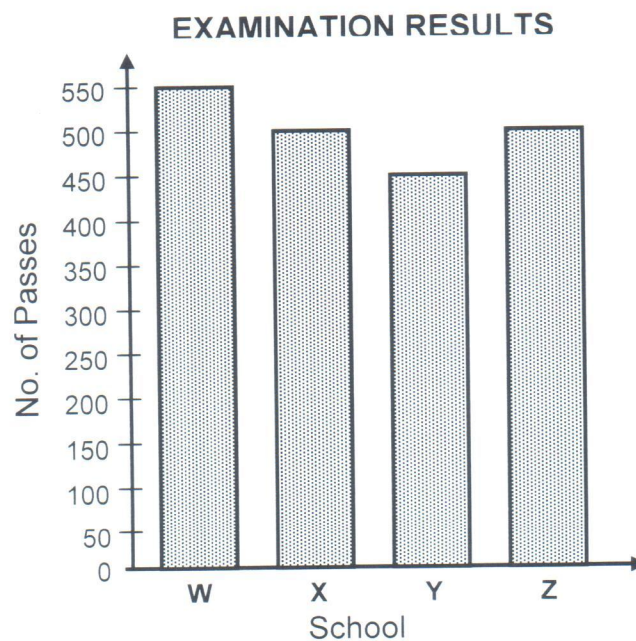
Looking at the first graph, the scale is sensible but the scale for the second graph (which is the same scale as the first graph) is too small.

- (d) Re-draw the graph in part (b) using a better scale.

See the graph on the next page.

JUDGES' POINTS IN A DIVING COMPETITION**Example 2**

Below is a column graph showing the number of passes in four schools in 2012.



Let us answer the following questions using the information shown in the above graph.

- (a) Which school had the highest number of passes?

Note that the highest or longest bar on the above graph is the one which is labeled **W** and which corresponds to the number of 550 passes.

Answer: W

- (b) What was the total number of passes in 2012?

We add all the number of passes which correspond with the height of the bars.

We have,

$$550 + 500 + 450 + 500 = 2000$$

Answer: 2000

- (c) Find the mean number of passes in 2012.

$$\text{Mean} = \frac{\text{Total number of passes}}{\text{Total number of schools}}$$

$$= \frac{2000}{4}$$

$$= 500$$

Answer: 500

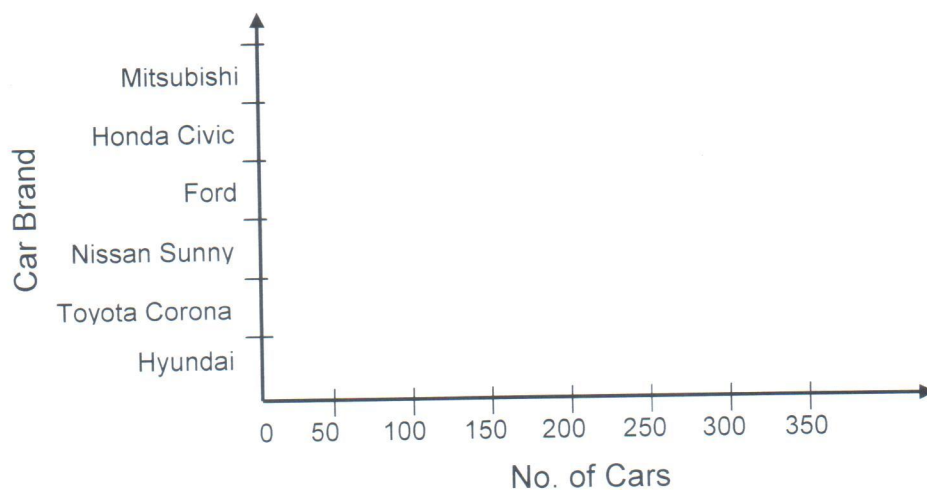
NOW DO PRACTICE EXERCISE 4

**Practice Exercise 4**

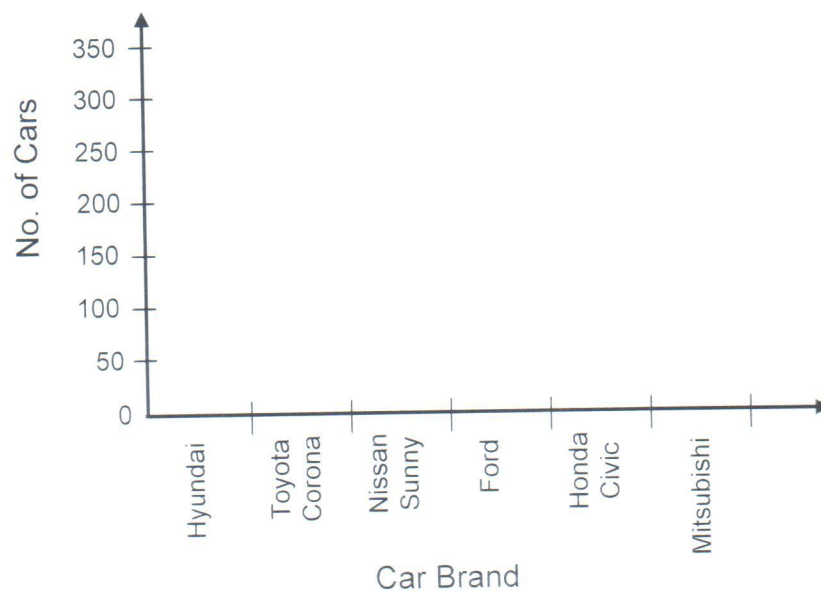
1. The table shows the different brand of cars registered in PNG in 2012.
- Draw: (a) a horizontal bar graph (b) a column graph to illustrate the data.

Car Brand	Number
Hyundai	300
Toyota Corona	350
Nissan Sunny	250
Ford	150
Mitsubishi	290
Honda Civic	195

- (a) **Cars Registered in PNG in 2012.**



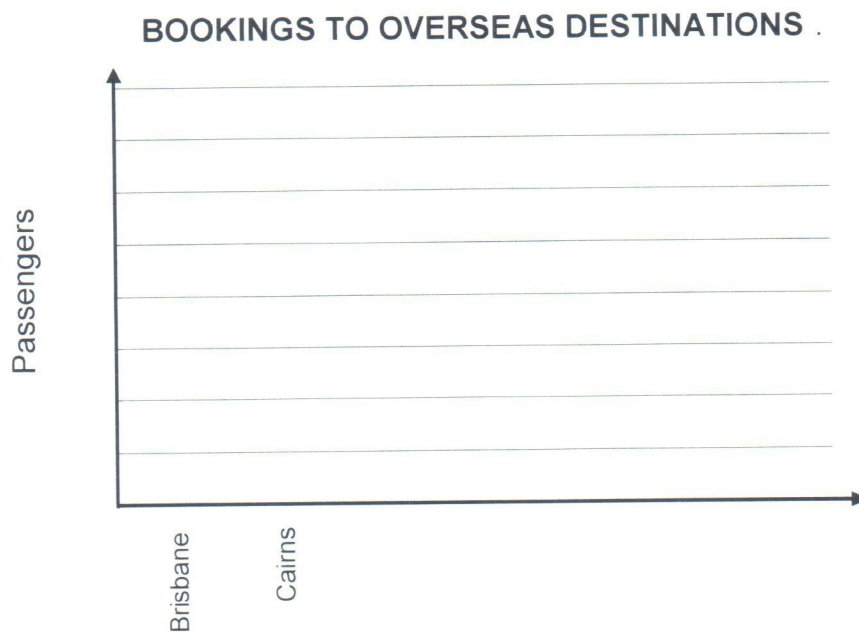
- (b) **Cars Registered in PNG in 2012.**



2. The bookings to overseas destinations on Air Nuigini are shown in the following table.

Destination	Passengers
Brisbane	80
Cairns	65
Hong Kong	45
Philippines	50
Singapore	30
Sydney	75

Complete the column graph below to illustrate the data.



2. Throughout 2013 Janet recorded the number of e-mails she sent to her many overseas pen friends each week. She posted 3 e-mails in the first week, 2 in the second week, none in the third week etc. Below is shown the complete number of e-mails she posted in each of the 52 weeks:

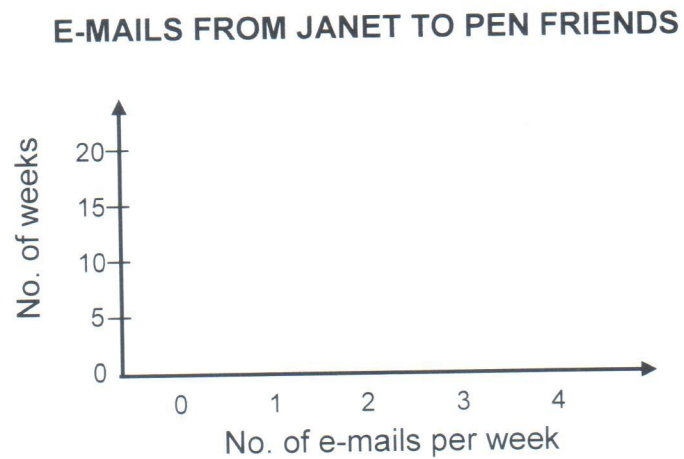
3,2,0,2,3,1,1,3,3,4,4,4,4,3,3,3,1,1,4,2,2,2,4,4,2,2

3,3,2,2,0,2,3,3,3,3,1,1,3,3,3,3,4,2,4,4,3,3,1,0,1

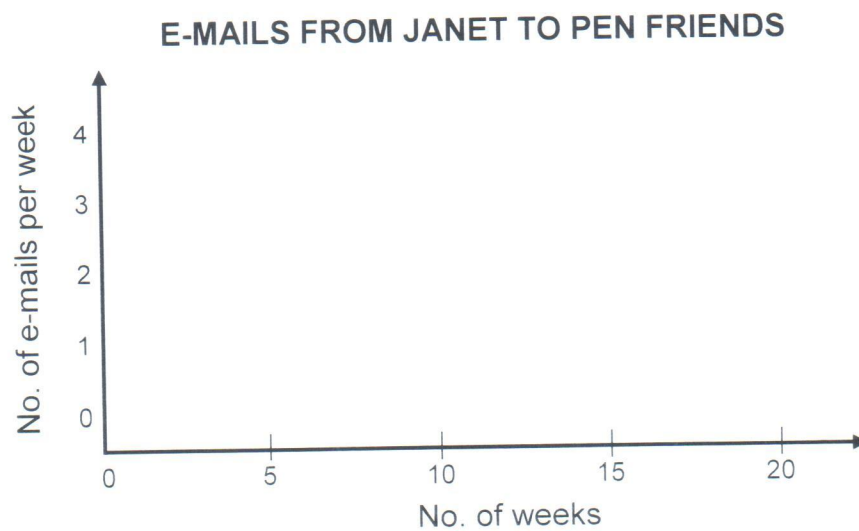
- (a) Complete the frequency table below showing this information.

Number of e-mails per week	Tally	Number of weeks (frequency) f
0		
1		

- (b) Complete the column graph below to show the above data.



- (c) Complete the horizontal bar graph below to show the above data.



4. Gabby was collecting data for research purposes. He stopped one hundred people in Vision City to ask them a number of questions. One question he asked them was their place of birth. He wrote the answers in the following table:

M	P	H	I	E	M	M	P	P	H
P	H	P	H	M	P	H	H	M	P
H	I	H	E	E	H	H	M	I	P
E	P	M	H	M	P	M	P	H	M
E	P	M	H	M	P	M	H	P	M
M	E	P	H	H	P	P	M	H	H
M	I	H	H	M	H	M	E	M	P
P	H	H	P	M	M	H	P	H	M
P	M	P	E	H	I	H	M	H	P
H	H	M	P	M	P	E	H	I	P

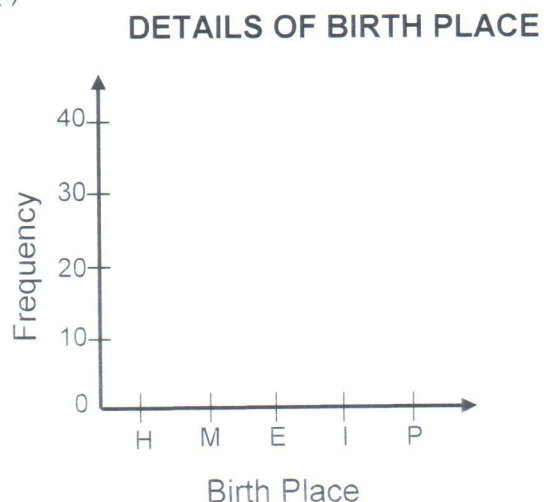
Key: H = Highlands P = Papua I = New Guinea Islands
M = Momase E = Expatriates

- (a) Use the data to draw a frequency table.

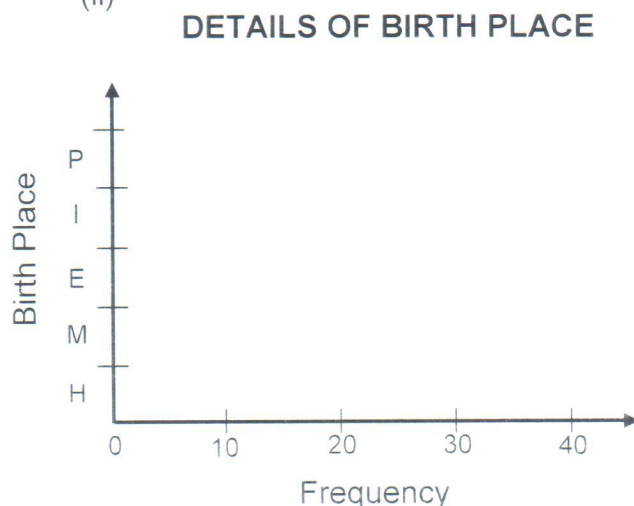
Birth Place	Tally Marks	Frequency
H		
M		
E		
I		
P		
Total		

- (b) Draw a: (i) column graph (ii) horizontal bar graph to illustrate the data.

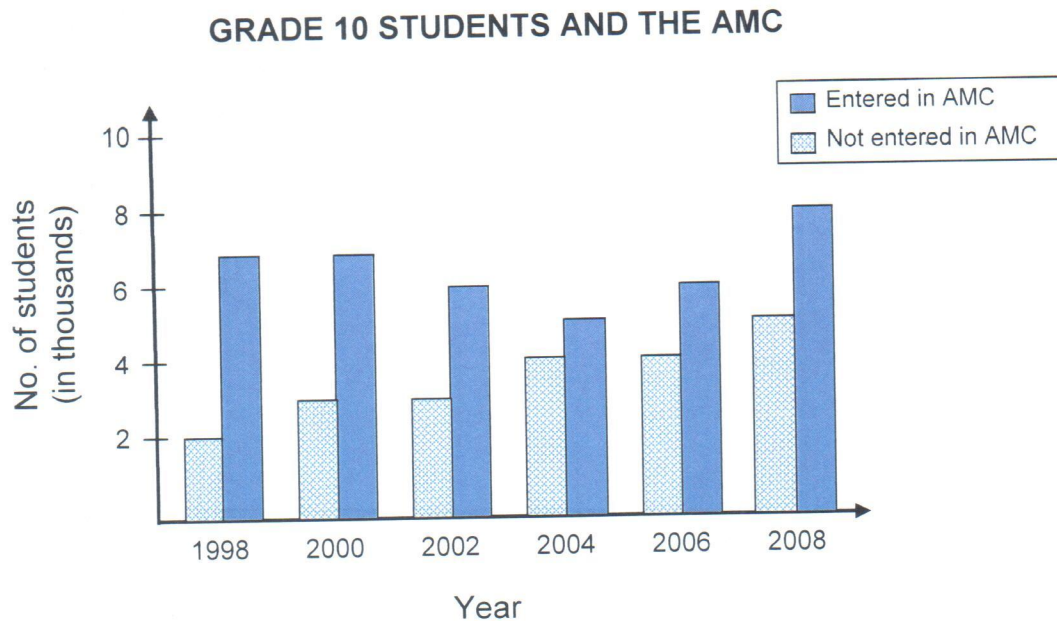
(i)



(ii)



5. The graph below shows the number of Grade 10 students entered and not entered in the Australian Mathematics Competition (AMC).



Study the above graph and then answer the following questions.

- (a) Write down the total number of students who entered in the AMC in the year 2008.

Answer: _____

- (b) How many Grade 10 students did not enter the competition in 1998?

Answer: _____

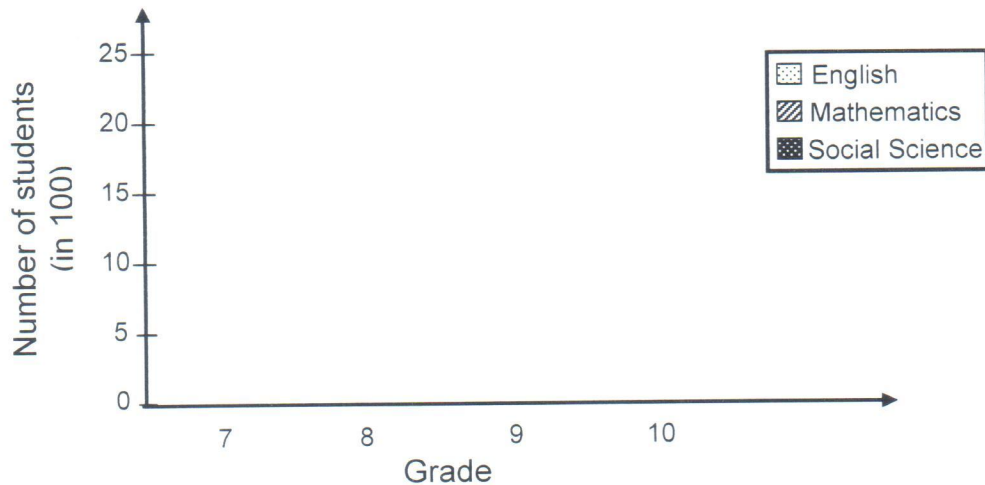
6. The table given below shows three subject completions at FODE during 2010.

Subject Completions	Grade 7	Grade 8	Grade 9	Grade 10
English	1500	500	750	2000
Mathematics	1000	750	1250	2000
Social Science	1000	500	1000	1500

(a) Complete the compound graphs below to illustrate the data by:

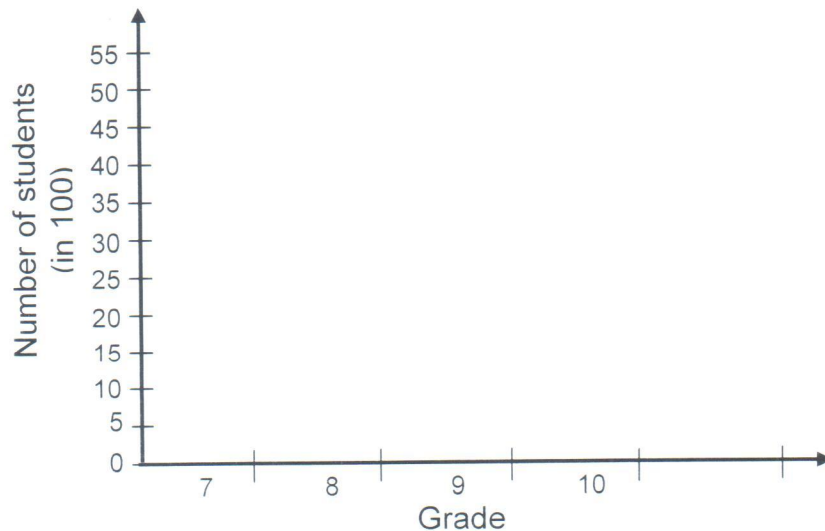
(i) Showing the subjects side by side.

SUBJECT COMPLETIONS AT FODE DURING 2010



(ii) Showing the subjects stacked, one above the other.

SUBJECT COMPLETIONS AT FODE DURING 2010



(b) Using either graph, find which grade had the lowest number of completions in 2010.

Answer: _____

(c) Using either graph, find which grade had the highest number of completions in 2010.

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 5: Pictograms and Pie Graphs



You have learned the range and the measures of central tendency in the previous lesson.



In this lesson, you will:

- differentiate between pictogram and pie graph
- draw, read and interpret pictogram and pie graphs.

As what we have said in the previous lesson, there are some other ways of representing data apart from using horizontal bar graphs, column graphs and compound graphs.

In this lesson, we will look at two of these other ways:

1. using pictograms
2. using pie graphs

You have already learned what pictograms and pie graphs are in your Grade 7 and Grade 8 Mathematics lessons. Let us look at their definition again.

Pictograms

Pictogram is a graph which uses pictures or symbols to represent and show statistical information.

Example

The following table shows how many cars were washed at the washing center of a service station at Vision City during four days of a week.

**CARS WASHED AT VISION CITY SERVICE STATION
DURING FOUR DAYS OF A WEEK**

Days	No. of cars washed
Monday	20
Tuesday	40
Wednesday	30
Thursday	25

After studying this table, we will draw a pictogram of the information given choosing

the symbol

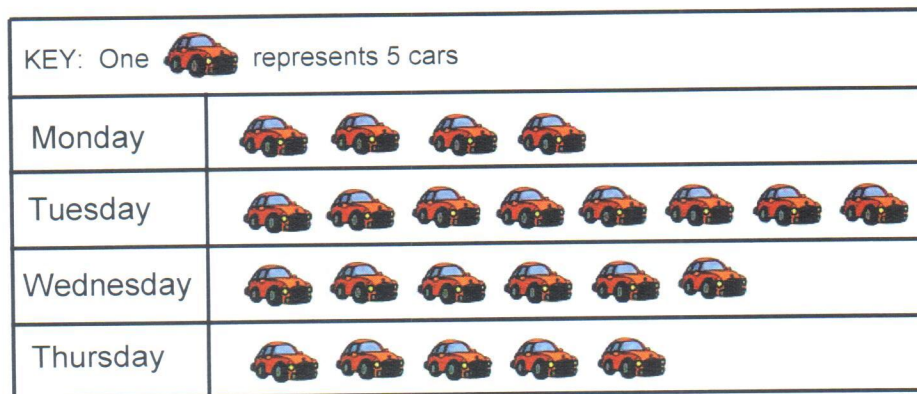


to represent the number of washed cars having the value of 5 cars.

The number of symbols required for each day is found by dividing each value by 5 as shown in the table below.

Monday 20 cars were washed, so number of symbols	$\frac{20}{5} = 4$
Tuesday 40 cars were washed, so number of symbols	$\frac{40}{5} = 8$
Wednesday 30 cars were washed, so number of symbols	$\frac{30}{5} = 6$
Thursday 25 cars were washed, so number of symbols	$\frac{25}{5} = 5$

Then we draw the pictogram:



Interpreting the pictograph

We can get information from the pictograph by answering the following questions:

- How many cars were washed on different days?

Answers: On Monday $4 \times 5 = 20$ cars,
 On Tuesday $8 \times 5 = 40$ cars,
 On Wednesday $6 \times 5 = 30$ cars,
 On Thursday $5 \times 5 = 25$ cars

- On which day were the maximum numbers of cars washed?

Answer: On Tuesday maximum numbers of cars washed were 40.

- On which day were the minimum numbers of cars washed?

Answer: On Monday: 20 cars

- How many more cars were washed on Tuesday than Wednesday?

Answer: $40 - 30 = 10$ more cars

- How many total cars were washed in four days?

Answer: $20 + 40 + 30 + 25 = 115$ cars

Pie Graphs

A Pie Graph or a Pie Chart is a circular chart which is divided into sectors in which the area of each sector represents the size of the data.

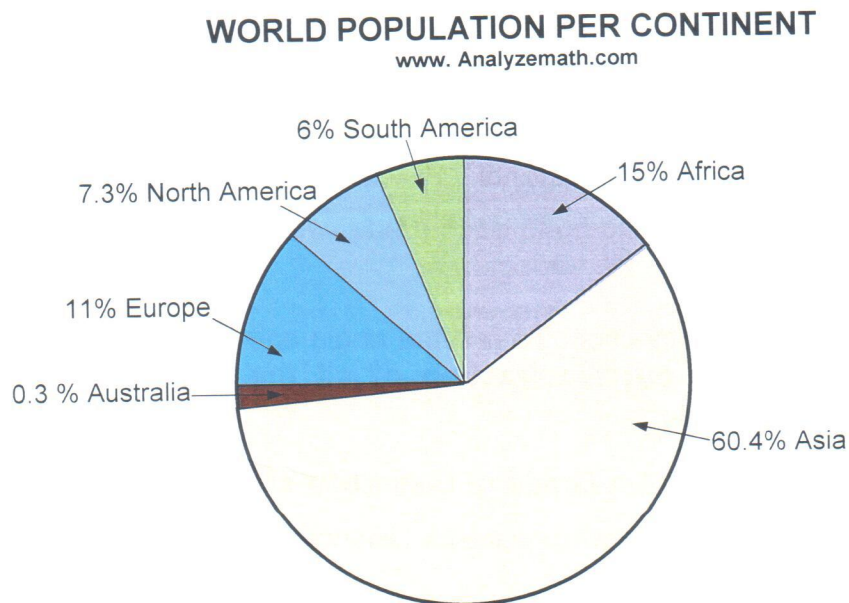
A pie graph or a pie chart is also known as **circle graph**. They are used to show data in proportion.

We often use pie graphs or pie charts when we want to show how a complete set of data is divided up. For example, the fractions of a person's wages spent on different items (rent, food etc.) could be appropriately represented using a pie chart.

A pie graph uses sectors of a circle to show the information.

Example

The pie graph below shows the percentages of the world population in each continent. The present world population is about 7 billion.



Interpreting the pie graph

We can get information from the pie graph by answering the following questions:

- a) How many people live in Africa?

Answer: $15\% \times 7 = 1.05$ billion

- b) How many people do not live in Asia then?

Answer: $7 - 60.4\% \times 7 = 2.772$ billion

- c) How many more people live in North America than in South America?

Answer: $(7.3\% - 6\%) \times 7 = 91$ million

Here is another example.

The table below shows the results of the examination of 200 medical students at the UPNG Medical School, Taurama Campus.

Grades	Frequency
Distinction	15
Credit	30
Upper Pass	60
Pass	90
Fail	5

To draw a pie graph of the given information above, we will need to work out the angles (how many degrees) for each "pie slice" (correctly called a **sector** of the pie graph) corresponding to the different Grades.

In a pie chart or graph, a full circle or one rotation has 360° , so we do the calculation by taking the following steps:

1. Work out the total number of students: $15 + 30 + 60 + 90 + 5 = 200$
2. To find the angle corresponding to each grade, work out the fraction of the total that got each grade.

Start with a grade of distinction. To find the angle corresponding to a Distinction grade, we first **work out the fraction of all the medical students who obtained a Distinction**.

Since 15 students gained a Grade of Distinction and there were 200 students altogether, then the fraction who gained a Distinction = $\frac{15}{200}$.

3. There are 360° in a full turn, so to work out the angle, **multiply the fraction by 360°** . Hence, $\frac{15}{200} \times 360^\circ = 27^\circ$

The grade Distinction has an angle of 27° .

4. Repeat this process to find the angle of the sectors for the other grades.
5. Once we have calculated the angles of the sectors, we can construct the pie graph.

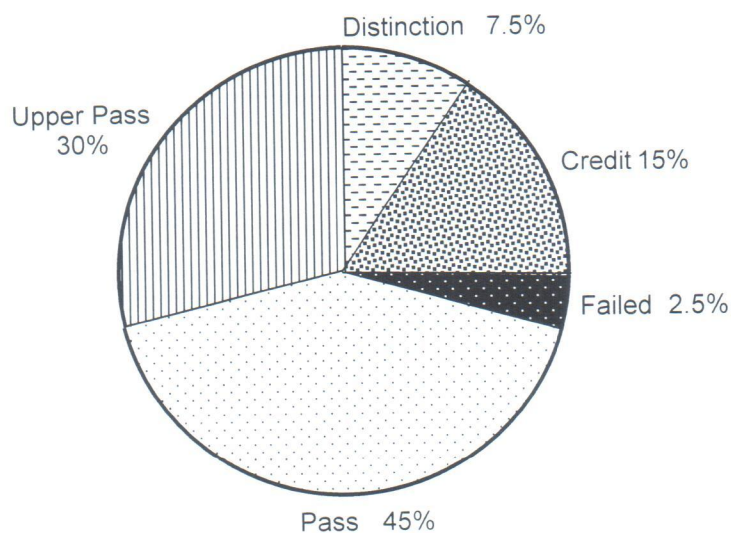
The following table on the next page shows the complete calculation of the angle of the sectors.

Calculation:

Grades	Frequency	Angle	Percentage
Distinction	15	$\frac{15}{200} \times 360^\circ = 27^\circ$	7.5 %
Credit	30	$\frac{30}{200} \times 360^\circ = 54^\circ$	15%
Upper Pass	60	$\frac{60}{200} \times 360^\circ = 108^\circ$	30%
Pass	90	$\frac{90}{200} \times 360^\circ = 162^\circ$	45%
Failed	5	$\frac{5}{200} \times 360^\circ = 9^\circ$	2.5%

Now, we can use the information in the table to draw the pie graph.

EXAMINATION RESULTS OF 200 MEDICAL STUDENTS



NOW DO PRACTICE EXERCISE 5



Practice exercise 5

1. Below is a pictograph showing the January to April average monthly rainfall of some places in Papua New Guinea during the year 2013.

JANUARY – APRIL 2013 AVERAGE MONTHLY RAINFALL(CM)

Place	Amount of rainfall in cm
Erave	          
Madang	           
Misima	         
Port Moresby	     
Rabaul	       
Sideia	      
KEY:  = 30 cm	

Refer to the pictograph to answer the following questions:

- (a) Give the information regarding the rainfall in each place in centimeters.

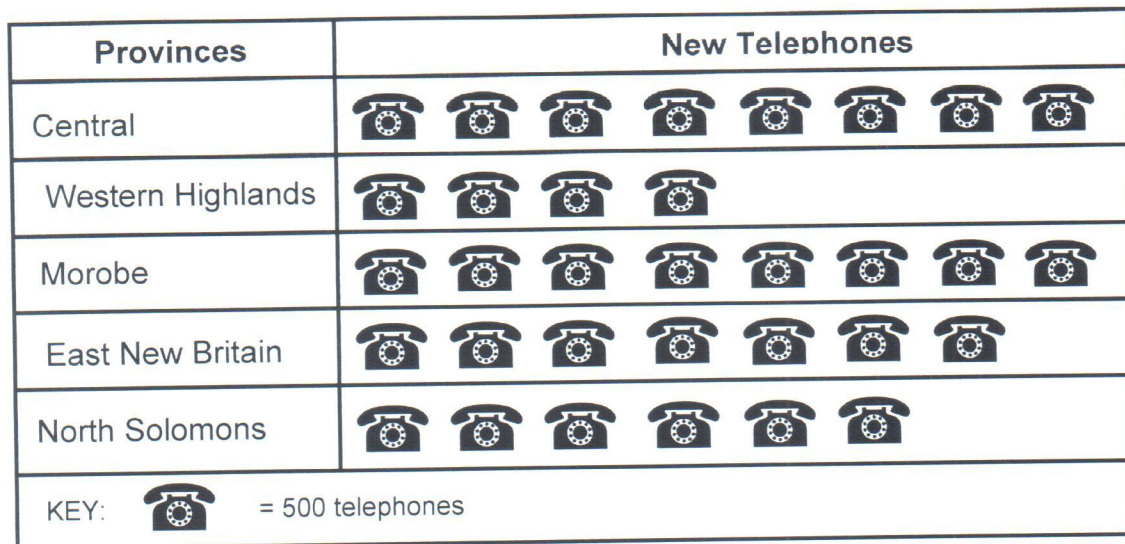
Erave	_____
Madang	_____
Misima	_____
Port Moresby	_____
Rabaul	_____
Sideia	_____

- (b) In which place was there maximum rainfall?

- (c) In which place was there minimum rainfall?

- (d) Erave recorded how much more rainfall than Sideia?

2. Below is a pictogram showing new telephone connections in five provinces in Papua New Guinea.



Answer the following questions about the pictogram:

- Which province/provinces had the highest number of new telephone connections in 2004?
- Which province/provinces had the least number of new telephone connections in 2004?
- How many new telephones were connected in North Solomons?
- How many new telephones altogether were connected in 2004?

3. Below is a table showing the number of female FODE students who obtained different Grades in an examination.

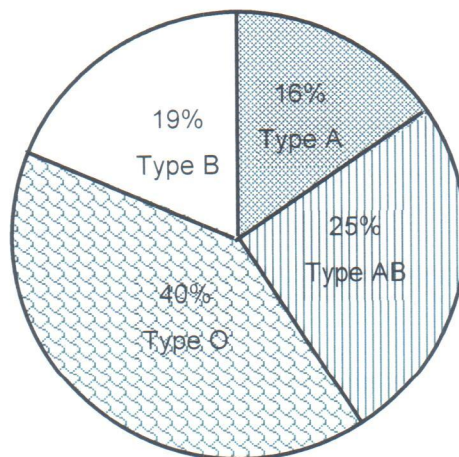
Grade	Frequency
Distinction	5
Credit	15
Upper Pass	25
Pass	50
Failed	5

Draw a pictograph of this information on the grid on the next page.

Use this KEY:  = 5 females

Grades	Number of Females
Distinction	
Credit	
Upper Pass	
Pass	
Failed	

4. The pie chart below shows the percentages of blood types for a group of 200 people.



BLOOD TYPES FOR A GROUP OF 200 PEOPLE

Answer the following questions based on the graph:

- How many people, in this group, have blood type AB?
- How many people, in this group, do not have blood type O?
- How many people, in this group, have blood types A or B?

5. Gina did a survey of her friends to find out which kind of movie they liked best.

Here are the results:

FAVORITE TYPE OF MOVIE

Movie Type	Frequency
Comedy	4
Action	5
Romance	6
Drama	1
Science Fiction	4

A pie chart is to be drawn showing this information.

- (a) Work out the angle sector for each favorite movie type.
- Comedy
 - Action
 - Romance
 - Drama
 - Science Fiction
- (b) Draw the pie graph showing the percentages of each movie type.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 6: Histograms and Cumulative Frequency Graphs



You have learnt the difference between pictograph and pie graph in the previous lesson. You have also learned how to read, interpret and draw pictograph and pie graph.



In this lesson, you will:

- distinguish between histograms, bar graphs and cumulative graphs
- read and interpret histograms and cumulative graphs
- construct histogram and cumulative frequency graphs.

As we have learned in Grade 9 there are several ways to collect and organize data. Two of these are the histogram and cumulative frequency graphs.

First in this lesson, we will learn the difference between histogram and a bar graph.

Like the pictogram and pie graph, **bar charts** and **histograms** are used to compare the sizes of different groups. Histograms and bar graphs look very similar but there is a difference between them.

Let us take a look at these two with an example of each.

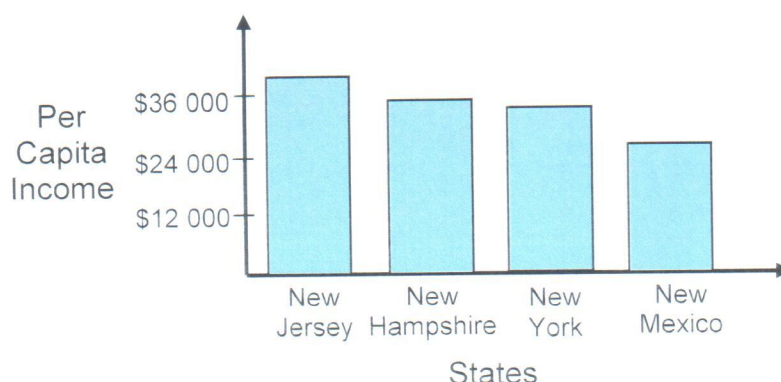
Bar Charts

A bar chart is made up of columns plotted on a graph. Here is how to read a bar chart.

- The columns are positioned over a label that represents a categorical variable.
- The height of the column indicates the size of the group defined by the column label.

The bar chart below shows average per capita income for the four "New" states: New Jersey, New York, New Hampshire, and New Mexico.

AVERAGE PER CAPITA INCOME FOR FOUR "NEW" STATES



As you can see in the bar graph above, the bars represent different categories. And if we wanted to re-arrange the bar we can do so. For example, we could move the “New Mexico” to go after “New Jersey” but before “new York”.

This means that, when we draw the bar graph the **order of the bars does not matter** at all as long as the thousands of dollars still match every category.

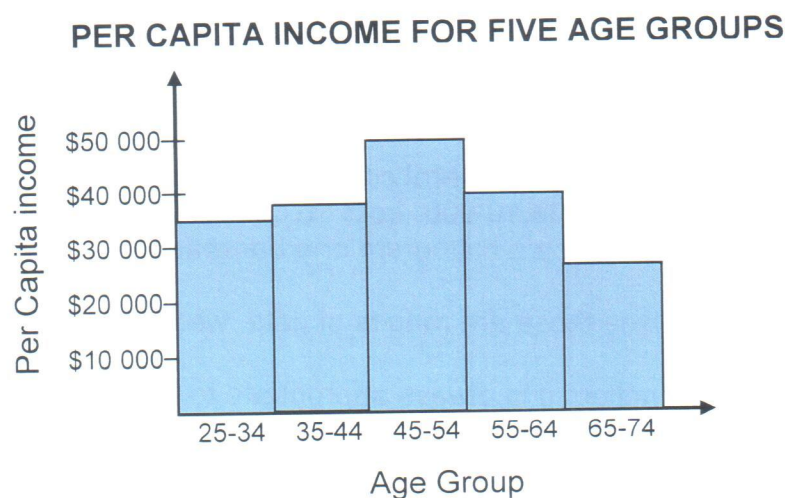
Now let us look at a Histogram.

Histograms

Like a bar chart, a histogram is made up of columns plotted on a graph. Usually, there is no space between adjacent columns. Here is how to read a histogram.

- The columns are positioned over a label that represents a **quantitative variable**.
- The column label can be a single value or a range of values.
- The height of the column indicates the size of the group defined by the column label.

The histogram below shows per capita income for five age groups.



As you can see, in the histogram, the bars do not really represent different categories anymore. The Histogram uses continuous data where the bars represent **ranges of data** (ages) rather than categories and the bars are drawn without gaps. And if we wanted to rearrange the bars, we cannot do so.

For example, we cannot move “65 – 74” to go after “25 – 34” but before “35 -44” because it would be too confusing.

This means that when we draw a histogram, the **order of the bars do matters** as the per capita income should correspond to the ranges of each age group.

On the next page is a summary of the three main differences between bar graphs and histograms.

Differences between a histogram and a bar graph

1. With bar graphs, each column represents a group defined by a categorical variable; and with histograms, each bars represents a group defined by a quantitative variable.
2. Histograms are used to show distributions of variables while bar charts are used to compare variables.
3. Bars can be reordered in bar charts but not in histograms.
4. Histogram is only used to plot the frequency of score occurrences in a continuous data set that has been divided into classes, called bins. Bar graphs, on the other hand, can be used for a great deal of other types of variables including ordinal and nominal data sets.

Now let us think about different sets of data and which would be better to use a histogram or a bar graph?

Example 1

Suppose we want to compare total revenues of five different companies. What would we use a histogram or a bar graph?

Answer: Of course we would use a bar graph where each company is a different category and gets its own bar to show total revenue.

Example 2

We have measured revenues of several companies. We want to compare numbers of companies that make from 0 to 10 000; from 10 000 to 20 000; from 20 000 to 30 000 and so on. What would we use a histogram or a bar graph?

Answer: Since those are ranges of data, we would use a histogram.

One effect of this distinction: it is always appropriate to talk about the **skewness** of a histogram; that is, the tendency of the observations to fall more on the low end or the high end of the horizontal axis.

With bar charts, however, the horizontal axis does not have a low end or a high end; because the labels on the horizontal axis are categorical - not quantitative. As a result, it is less appropriate to comment on the skewness of a bar graph.

Frequency Histogram

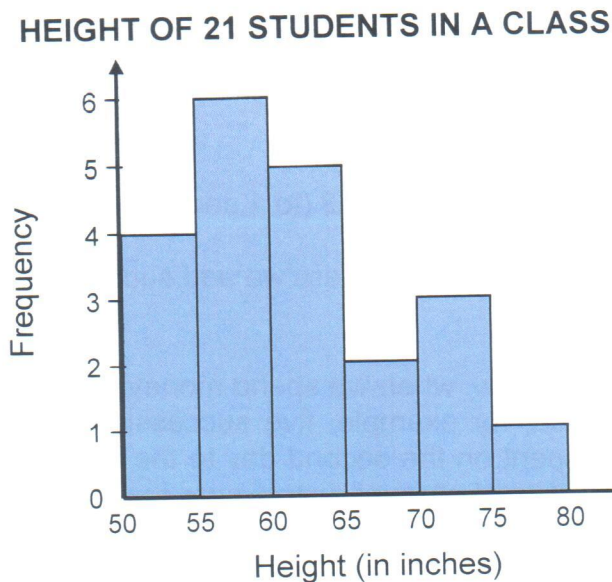
A Frequency Histogram is a special histogram that uses vertical columns to show frequencies (how many times each score occurs):

Example 1

The histogram on the next page shows the heights of 21 students in a class, grouped into 5-inch groups.

Study the histogram and answer the question that follows.

Here is the histogram.



- (a) How many students were greater than or equal to 60 inches tall?

Answer:

The number of students that were greater than or equal to 60 inches tall are shown in the bars representing the groups 60 – 65, 65 – 70, 70-75 and 75-80.

$$= 5 + 2 + 3 + 1$$

$$= 11$$

- (b) How many students were greater than or equal to 55 inches tall but less than 70 inches tall?

Answer:

The number of students that were greater than or equal to 55 inches tall but less than 70 inches tall are shown in the bars representing the groups 55 - 60, 60 - 65 and 65 – 70.

$$= 6 + 5 + 2$$

$$= 13$$

- (c) Draw a frequency distribution table including an **fx** column and then calculate the mean score (to 1 decimal place).

HEIGHTS OF 21 STUDENTS IN A CLASS			
Age Group	Frequency(f)	Midpoints(x)	fx
50 – 55	4	52.5	210
55 – 60	6	57.5	345
60 – 65	5	62.5	312.5
65 – 70	2	67.5	135
70 – 75	3	72.5	217.5
75 - 80	1	77.5	77.5
N= 21			Σfx= 1297.5

$$\begin{aligned}\text{Mean} &= \frac{\sum fx}{N} \\ &= \frac{1297.5}{21} \\ &= \mathbf{61.8} \text{ (to 1 decimal place)}\end{aligned}$$

There are many examples in everyday life where we add successive amounts to obtain a total.

For example, we often want to know, when we spend money each day, how much we spend altogether over a period, for example, five successive days. To do this, we need to add the amount we spent on the second day to the amount we spent on the previous day, thus giving us a total amount spend over the first two days. We then add on this total the amount we spent on the third day, giving us a new total and so on.

In Grade 9 we learned that these successive totals are called as **running totals** or **cumulative totals**. The word “cumulative” came from the word “accumulate” which means “to gather together” or “to store up” by adding items one by one.

Example 1

The amounts spent each day by Steven for days (Mondays to Friday) are recorded in the table below.

Day	Amount Spent
Monday	K8
Tuesday	K10
Wednesday	K4
Thursday	K5
Friday	K7

Let us now extend the above table to include a “cummulative total” column as shown below.

To work out the cumulative totals, just add up as you go. The first value in the “Cummulative Total” column (K8) is the same as the first value in the “Amount Spent” column. The next cummulative total is obtained by adding the amount spent on Tuesday (K10) to the amount spent on Monday: $K8 + K10 = K18$

Day	Amount Spent	Cummulative Total
Monday	K8	K8
Tuesday	K10	K18

Continuing with the same way for succeeding days we finally obtain the cummulative total table on the next page.

Day	Amount Spent	Cumulative Total
Monday	K8	K8
Tuesday	K10	K18
Wednesday	K4	K22
Thursday	K5	K27
Friday	K7	K34

The last cumulative total should match the total of the amount spent.

K34 is the last cumulative total...

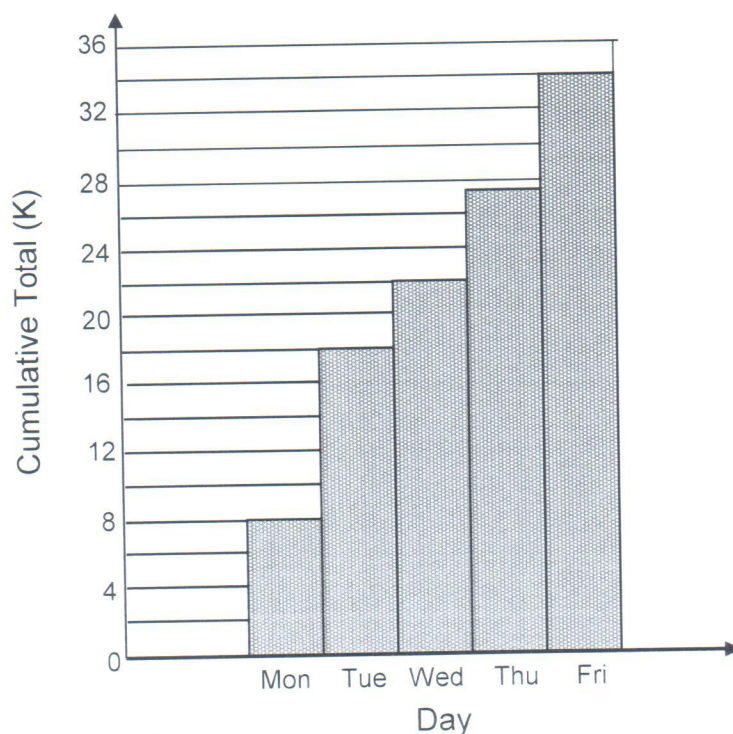
It is also the total of all the amounts spent.

$$K8 + K10 + K4 + K5 + K7 = \mathbf{K34}$$

So, we got it right.

Using the data in the cumulative total table, we can now present and draw the cumulative total graph. Successive cumulative totals are always shown on the vertical axis of the graph.

CUMULATIVE TOTAL GRAPH OF MONEY SPENT



Notice that from the above cumulative total graph:

- (a) each column is never longer than (though it can be equal) the one its left, so it generally steps upwards to the right.
- (b) the height of the last column gives the final cumulative total.

Cummulative Frequency

Recall that the frequency of an event tells us how often the event occurs.

For example, a dice is rolled fourteen times and the score on the dice recorded after each roll is shown below.

2, 3, 4, 1, 3, 3, 6, 4, 1, 3, 2, 2, 3, 5

Thus the frequency of a score of 1 is 2; the frequency of a score of 2 is 3 and so on. We can now use this information to draw a frequency table showing how often the scores 1, 2, 3, 4, etc. occur.

Score	Frequency
1	2
2	3
3	5
4	2
5	1
6	1

We can now use the above frequency table to find the cumulative frequency of each score and draw the cumulative frequency table.

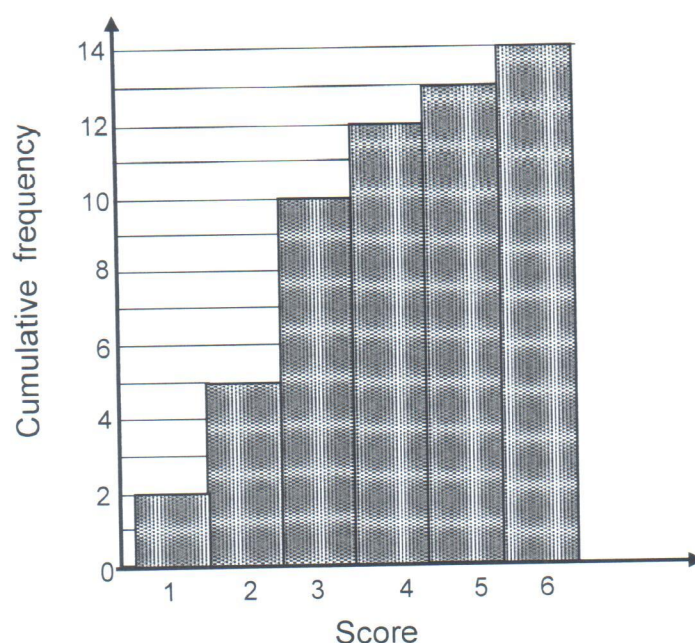
Cummulative frequency is the total frequency up to a given data value.

The cumulative frequency of each score is found by adding the frequencies of all the scores up to and including that particular score.

Score	Frequency	Cummulative Frequency
1	2	2
2	3	5
3	5	10
4	2	12
5	1	13
6	1	14

Now we can draw the cumulative frequency graph. We use the term “histogram” rather than “graph” because we will mark the score, which of course consists of numbers, on the horizontal axis and we will mark the cumulative frequency on the vertical axis - similar to what we did when we drew the frequency histogram at the start of our lesson.

CUMMULATIVE FREQUENCY HISTOGRAM OF A



Notice in the above cumulative frequency histogram, as with all cumulative total graphs, it generally steps upward to the right. The height of the last column gives the final cumulative frequency.

Example 2

Draw a cumulative frequency histogram for the frequency table below showing the raw scores from a SAT Mathematics test for 150 Grade 10 students. The highest score possible was 1000 and the lowest score was 100.

Scores	Frequency
100 – 200	1
200 – 300	4
300 – 400	10
400 – 500	8
500 – 600	15
600 – 700	55
700 – 800	44
800 – 900	10
900 – 1000	3

To draw the cumulative frequency histogram we need to construct the cumulative frequency table by extending it to include cumulative frequency column as shown on the next page.

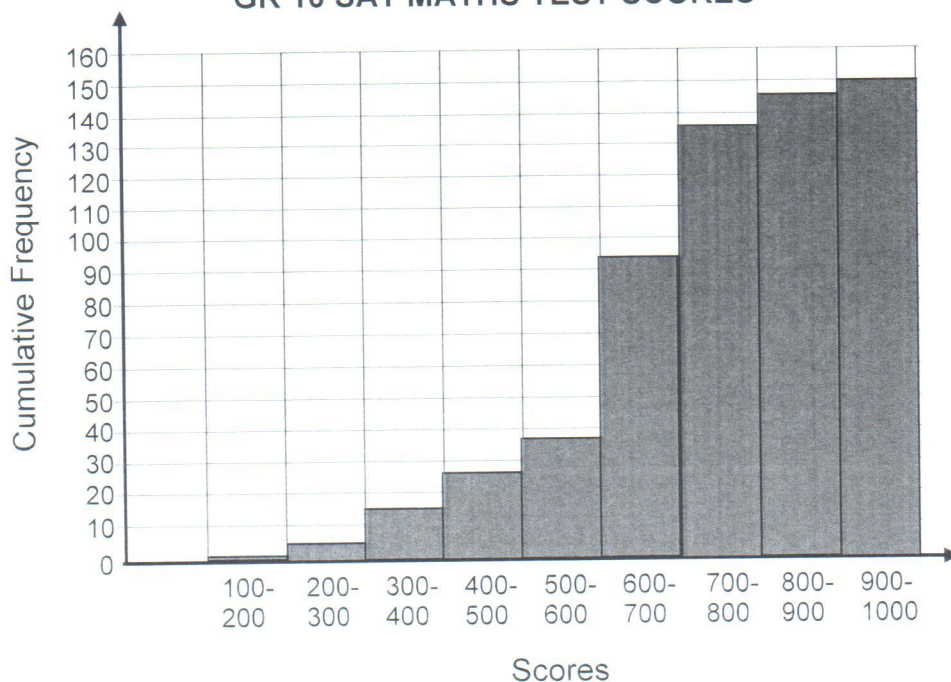
The first value in the cumulative frequency column is equal to the first value in the frequency column (1). Successive cumulative frequencies are obtained by adding the frequency in the second row ($1 + 4 = 5$); then to this new cumulative frequency (5) is added the frequency in the third row and so on.

**CUMULATIVE FREQUENCY TABLE
GR 10 SAT MATHS TEST SCORES**

Scores	Frequency	Cummulative frequency
100 – 200	1	1
200 – 300	4	5
300 - 400	10	15
400 – 500	8	23
500 – 600	15	38
600 – 700	55	93
700 – 800	44	137
800 – 900	10	147
900 - 1000	3	150

Now we can draw the cumulative frequency histogram. We will mark the cumulative frequency on the vertical axis and the scores on the horizontal axis.

**CUMULATIVE FREQUENCY HISTOGRAM
GR 10 SAT MATHS TEST SCORES**



Notice in the above cumulative frequency histogram, as with all cumulative total graphs, it generally steps upward to the right. The height of the last column gives the final cumulative frequency.

Using the cumulative frequency, answer the following questions.

- (a) Which score intervals has the greatest range?

Answer: **600- 700**

- (b) Which score intervals has the lowest range?

Answer: **100- 200**

Example 3

A school gave a final examination to all students taking Social Science. The number of students taking the examination was 180. The test grades for these students were grouped into a table shown below.

Scores	Frequency
51 – 60	15
61 – 70	30
71 – 80	55
81 – 90	50
91 – 100	30

- (a) How many students scored “60 or less” (51 – 60) on the test?
 (b) How many students scored “70 or less” (51 – 70) on the test?
 (c) How many students scored “80 or less” (51 – 80) on the test?
 (d) How many students scored “90 or less” (51 – 90) on the test?
 (e) How many students scored “100 or less” (51 – 100) on the test?

The answers to all the above questions, are found by cumulating the frequencies on the above table.

Scores	Frequency	Scores	Cumulative Frequency
51 – 60	15	51 – 60	15
61 – 70	30	51 – 70	45
71 – 80	55	51 – 80	100
81 – 90	50	51 – 90	150
91 – 100	30	51 – 100	180

Hence we have the following as answers:

- (a) The number of students who scored “60 or less” on the test is **15**.
 (b) The number of students who scored “70 or less” on the test is **45**.
 (c) The number of students who scored “80 or less” on the test is **100**.
 (d) The number of students who scored “90 or less” on the test is **150**.
 (e) The number of students who scored “100 or less” on the test is **180**.

The Cumulative Frequency Curve or Ogive

For continuous data, the cumulative frequency is the total frequency up to a given class boundary or class limit.

The cumulative frequency graph is plotted using the cumulative frequency against the lower or upper class boundary of each interval and join the points with a smooth curve. We call this graph or curve “the **Ogive**” or the “**Cumulative frequency curve**”

There are two types of ogive or cumulative frequency curve.

1. **The less than ogive**
2. **The greater than ogive**

Steps for constructing a Less than Ogive (Less than Cumulative Frequency Curve):

1. Draw and label the horizontal and vertical axes.
2. Take the cumulative frequencies along the y axis (vertical axis) and the upper class limits on the x axis (horizontal axis).
3. Plot the cumulative frequencies against each upper class limit.
4. Join the points with a smooth curve.

Steps for constructing a Greater than Ogive (Greater than Cumulative Frequency Curve):

1. Draw and label the horizontal and vertical axes.
2. Take the cumulative frequencies along the y axis (vertical axis) and the lower class limits on the x axis (horizontal axis)
3. Plot the cumulative frequencies against each lower class limit.
4. Join the points with a smooth curve.

Example

Draw a cumulative frequency graph for the frequency table below.

Length (mm)	Frequency
11 – 15	2
16 – 20	4
21 – 25	8
25 – 30	14
31 – 35	6
36 – 40	4
41 – 45	2

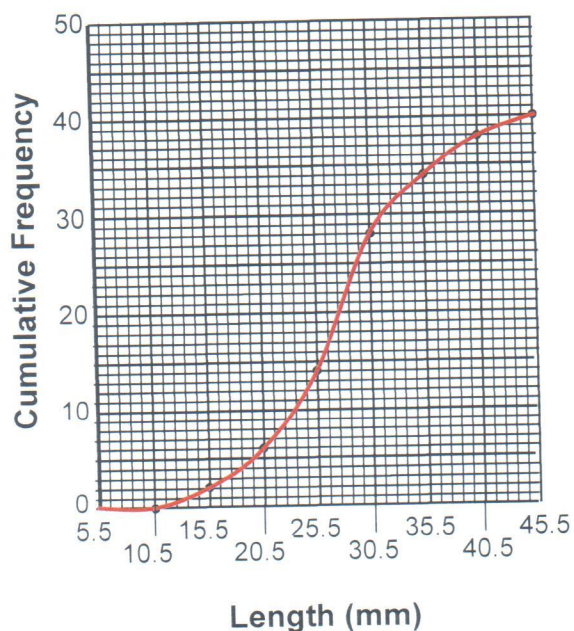
Solution:

We need to add a class with 0 frequency before the first class and then find the lower and upper class boundary for each class interval.

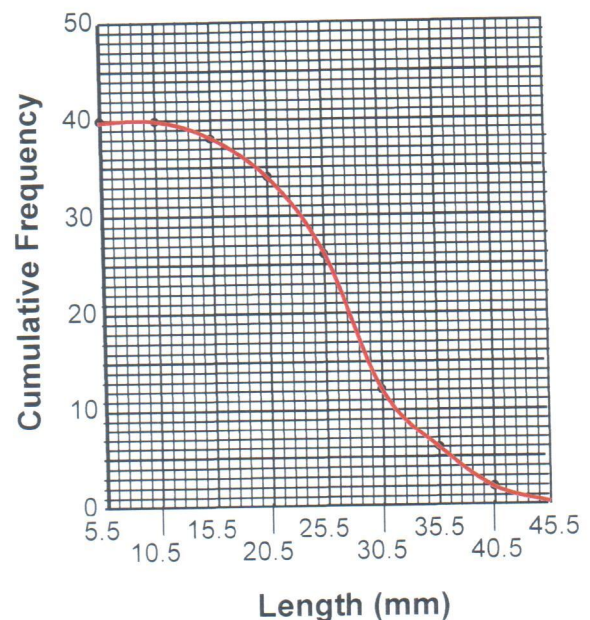
Length (mm)	Frequency	Class Limits or Boundaries		Cumulative Frequencies(Ogives)	
		Lower	Upper	Less than	Greater than
6 - 10	0	5.5	10.5	0	40
11 - 15	2	10.5	15.5	2	40
16 - 20	4	15.5	20.5	6	38
21 - 25	8	20.5	25.5	14	34
25 - 30	14	25.5	30.5	28	26
31 - 35	6	30.5	35.5	34	12
36 - 40	4	35.5	40.5	38	6
41 - 45	2	40.5	45.5	40	2

Now let us draw the Ogives.

The Less than Ogive



The Greater Than Ogive

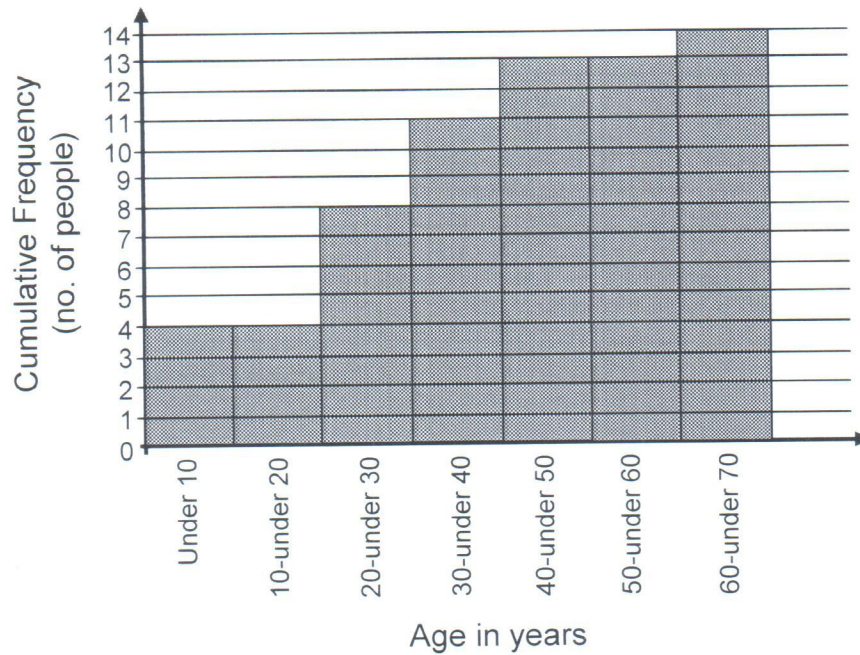


NOW DO PRACTICE EXERCISE 6



Practice Exercise 6

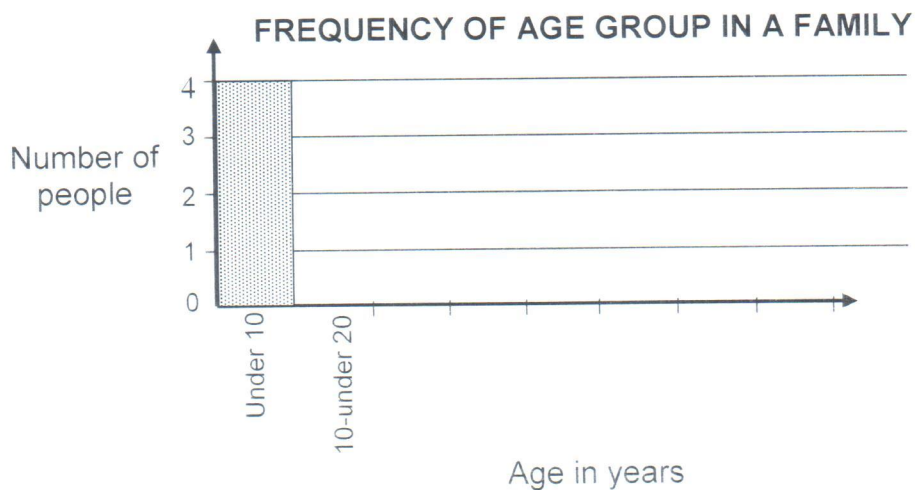
1. Below is a cumulative frequency histogram of the number of people in different age groups in a large family.



- (a) Using the cumulative histogram, complete the frequency table.

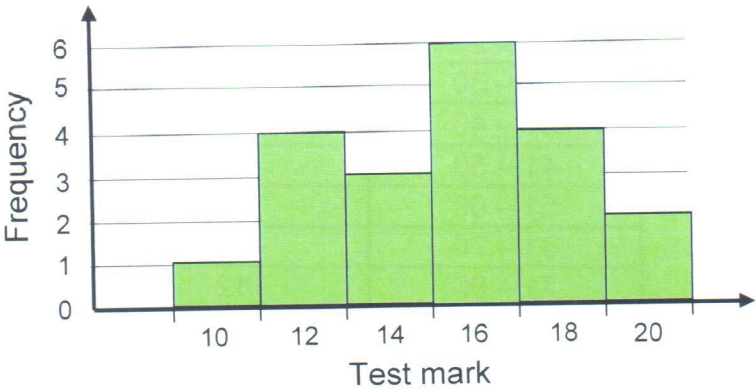
Age Group	Frequency
under 10	
10 – under 20	

- (b) Complete the frequency graph below of the above data.



2. Study this Frequency Histogram, which gives a set of Mathematics Test Marks.

FREQUENCY HISTOGRAM OF MATHS MARKS

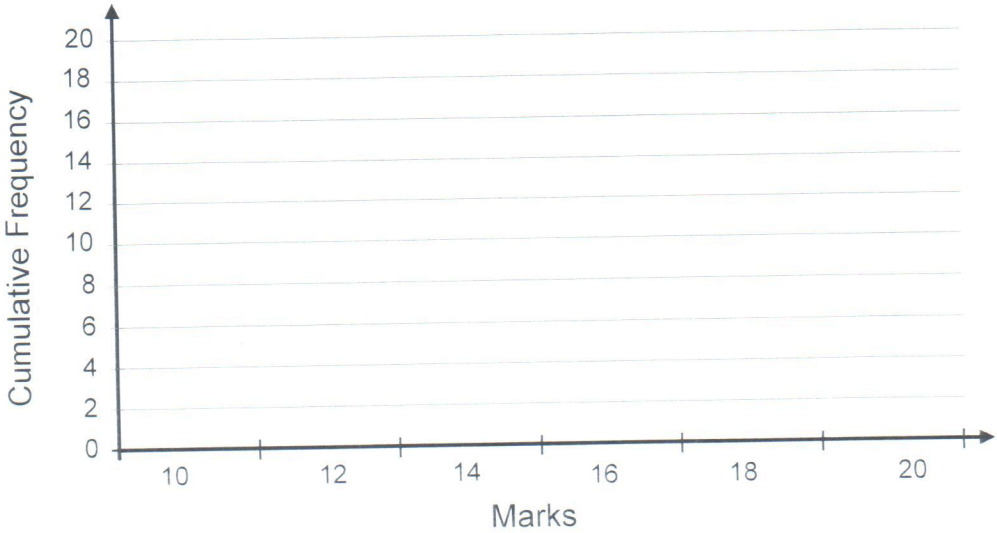


(a) Using the Frequency Histogram above complete this Cumulative Frequency table.

Marks	Frequency	Cummulative Frequency
10	1	

(b) Draw the Cumulative Frequency Histogram showing the data in your table.

CUMMULATIVE FREQUENCY HISTOGRAM OF MATHEMATICS MARKS



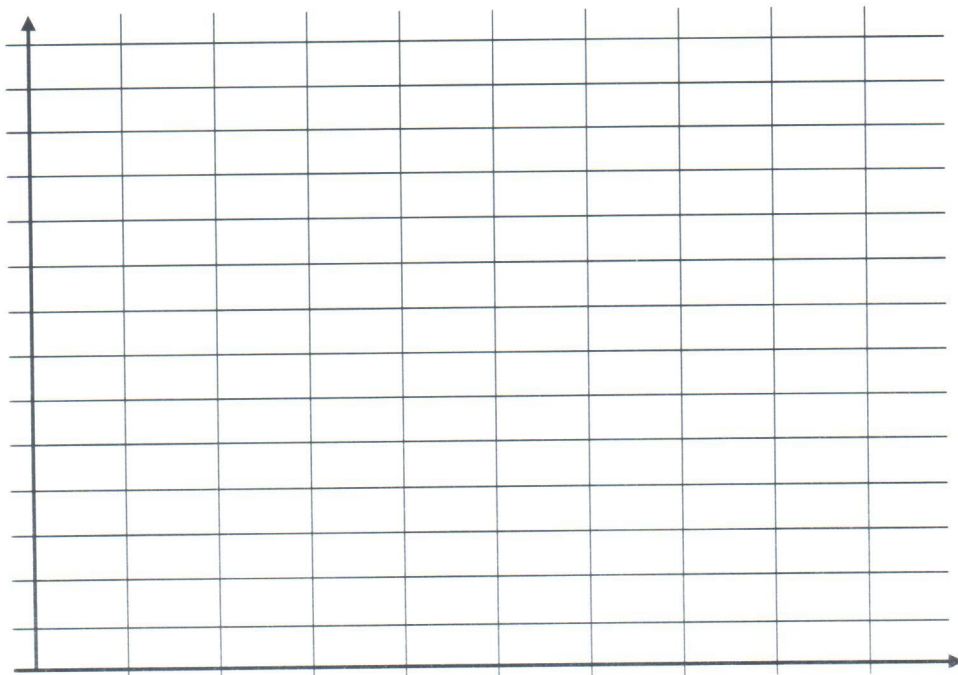
3. Given the frequency table of different heights in centimetres below.

Heights (cm)	Frequency
11 – 20	3
21 – 30	5
31 – 40	7
41 – 50	8
51 – 60	10
61 – 70	15
71 – 80	12
81 – 90	6
91 – 100	4

- (a) Construct a less than cumulative frequency table.

Heights (cm)	Frequency	Upper Class boundaries	Cumulative frequencies
11 – 20	2		
21 – 30	3		
31 – 40	7		
41 – 50	8		
51 – 60	10		
61 – 70	15		
71 – 80	10		
81 – 90	6		
91 – 100	4		

- (b) Draw the ogive.



4. The table below shows a cumulative frequency distribution of runner athlete's ages.

**CUMMULATIVE FREQUENCY DISTRIBUTION
OF RUNNER ATHLETE'S AGES**

Age group	Cummulative frequency
20 – 29	8
20 – 39	18
20 – 49	25
20 – 59	31
20 – 69	35

- (a) Based on the table, how many runners are in their forties?
- (b) How many runners are in their forties?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

TOPIC 1: SUMMARY



This summarizes the important concepts and ideas to be remember.

- **Discrete data** consists of separate things that can be counted.
- **Continuous data** cannot be counted, only measured.
- **Class limits** sometimes spoken as **class boundaries** are the boundaries between one class and the next. Each class has an upper class limit and a lower class limit.
- **Lower class limit or boundary** is the number on the left of each individual class interval.
- **Upper class limit or boundary** is the number on the right of each individual class interval.
- **Class Width** is the difference between two consecutive lower limits or two consecutive upper limits. It is also determined by dividing the range by the number of classes.
- **Estimated mean** of Continuous data is computed by the formula:

$$\bar{X} = \frac{\sum fM}{N}$$

- **Estimated Median** of Continuous data is computed by the formula:

$$\text{Median} = L + \frac{\left(\frac{N}{2}\right) - F_b}{f_m} \text{ (i)} \quad \text{or} \quad \text{Median} = U - \frac{\left(\frac{N}{2}\right) - F_a}{f_m} \text{ (ii)}$$

- **Estimated Mode** of Continuous data is computed by the formula:

$$\text{Estimated Mode} = L + \frac{F_1 - F_0}{2F_1 - F_1 - F_2} \text{ (i)}$$

- **Range of grouped continuous data** is the difference between the upper limit of the top class and the lower limit of the bottom class.
- **Bar graph** is a graph that uses separate bars to show information. There are two types of bar graph: the column or vertical bar graph and the horizontal bar graph.
- **Compound graph** is a graph which can be drawn either by different bars beside each other or different bars on top of each other that are stacked one on top of another.
- **Histogram** is a special bar graph but each bar are drawn side by side without gaps and represents a range of data.
- **Cummulative frequency table** is a statistical method that has a column called cummulative frequency which tally's up the values of the frequencies.
- **Cummulative frequency graph** is a graph plotted from a cummulative frequency table. It shows the cummulative totals of a set of data to each point on the graph.

REVISE LESSONS 1-6. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 4.

ANSWERS TO PRACTICE EXERCISES 1 – 6

Practice Exercise 1

1. (a) continuous (g) discrete
(b) discrete (h) discrete
(c) continuous (i) continuous
(d) discrete (j) discrete
(e) continuous (k) discrete
(f) continuous (l) discrete

2. Joshua, Jemjem and Daniel

3. 0.5, 0.7, 0.9, and 1.2

4. (a) discrete
(b)

Number of costumers	Frequency
20	2
21	7
22	4
23	3
24	5
25	2
26	2

(c) 21

5.

Weights of Students (kg)	Frequency
43.5 – under 45.5	5
45.5 – under 47.5	6
47.5 – under 49.5	7
49.5 – under 51.5	9

6. (a) discrete

(b)

Fortnightly Salary	Tally Marks	Frequency (f)
420 - < 440	I	1
440 - < 460		5
460 - < 480	- -	14
480 - < 500	-	9
500 - < 520	-	8
520 - < 540	-	8
540 - < 560		5
560 - < 580	I	1
Total		50

Practice Exercise 21. (a) Range = $18 - 13 = 5$

(b) Class width = 1

(c) 13 – under 14

(d)

Length (km)	Frequency
13– under 14	5
14 – under 15	11
15 – under 16	12
16 – under 17	11
17 – under 18	1
Total	40

2.

Distance (km)	Frequency
1 – under 3	10
3 – under 5	7
5 – under 7	3
Total	20

3. (a) Range = $438 - 318 = 120$
 (b) Class width = $120 \div 7 = 17.14 = 18$
 (c)

Height (in cm)	Frequency
318 – under 336	4
336 – under 354	5
354 – under 372	2
372 – under 390	3
390 – under 408	2
408 – under 426	3
426 – under 444	1
Total	20

- (d) The class interval in which the weight increase maximum is 336 – under 354. [Frequency is more for the class 336 – under 354.]

Practice Exercise 3

1. (a) 13 (b) 15.5 (c) 20
2. (a)

Age Group	Frequency (f)	Class centers (M)	fM
1 – under 11	400	6	2400
11 – under 21	800	16	12 800
21 – under 31	2600	26	67 600
31 – under 41	4600	36	165 600
41 – under 51	4000	46	184 000
51 – under 61	2200	56	123 200
61 – under 71	1400	66	92 400
	N=16000		$\Sigma fM = 648000$

- (b) Estimated mean = 40.5
 (c) Median Class: 31 – under 41
 (d) Modal Class: 31 – under 41
 (e) 70
 (f) 23.75 %

3. (a)

Distance in Km	Frequency (f)	Midpoints (M)	Frequency x Midpoints (fM)
1 – under 4	18	2.5	45
4 – under 7	22	5.5	121
7 – under 10	16	8.5	136
10 – under 13	15	11.5	172.5
13 – under 16	8	14.5	116
16 – under 19	12	17.5	210
19 – under 22	6	20.5	123
22 – under 25	3	23.5	70.5
N = 100		$\Sigma fM = 994$	

- (b) i. 4 – under 7
 ii. 9.94 km
 iii. 71%

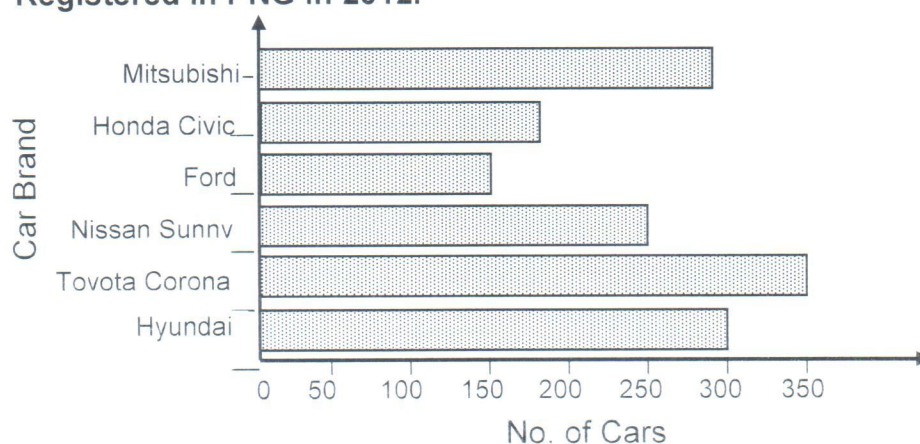
4. (a)

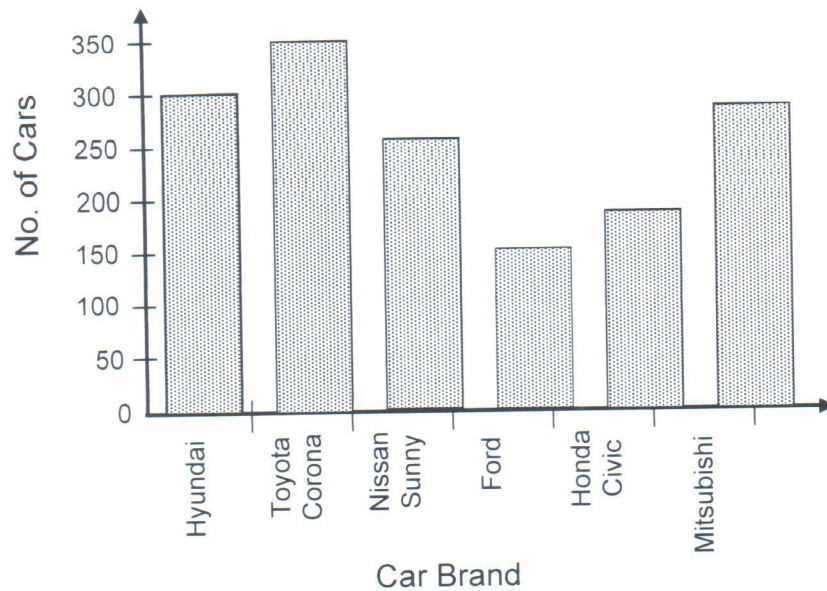
Height (cm)	Frequency (f)	Midpoints (M)	f x M (fM)	Class limits	
				Lower	Upper
140 – 144	2	142	284	139.5	144.5
145 – 149	4	147	588	144.5	149.5
150 – 154	3	152	456	149.5	154.5
155 – 159	4	157	628	154.5	159.5
160 – 164	5	162	810	159.5	164.5
165 – 169	5	167	835	164.5	169.5
170 – 174	3	172	516	169.5	174.5
175 – 179	4	178	712	174.5	179.5
N = 30		$\Sigma fM = 4829$			

- (b) i. class size = 5
 ii. 179
 iii. 140
 iv. 39
 v. NO
 vi. 161

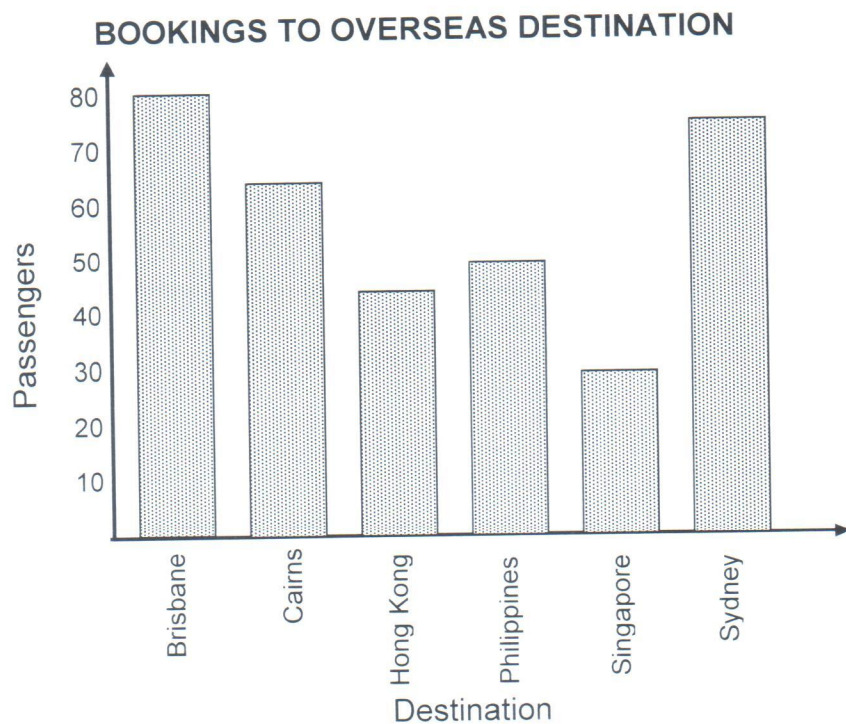
Practice Exercise 4

1. (a) Registered in PNG in 2012.



(b) **Cars Registered in PNG in 2012.**

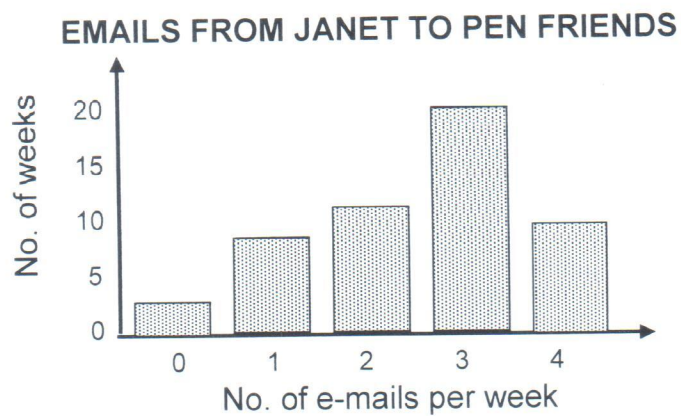
2.



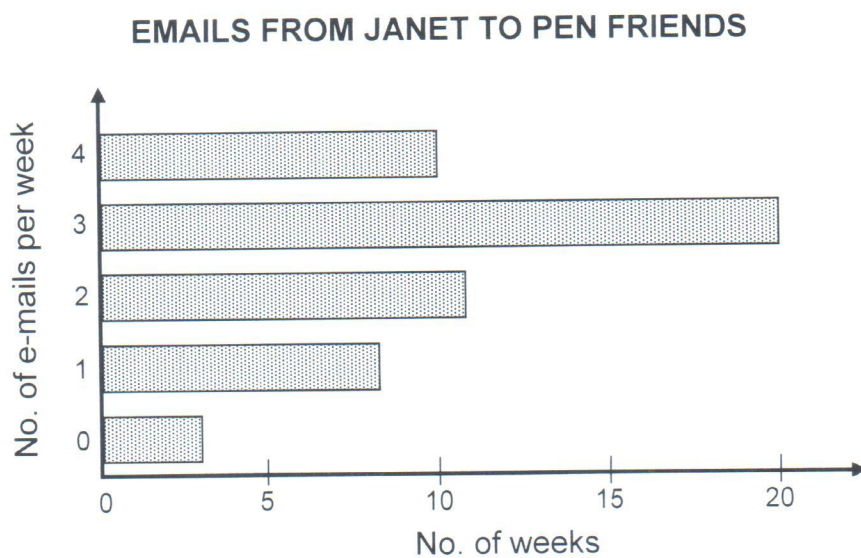
3. (a) Complete the frequency table below showing this information.

Number of e-mails per week	Tally	Number of weeks (frequency) f
0	///	3
1	/	8
2	//	11
3	/	20
4	////	10

(b)



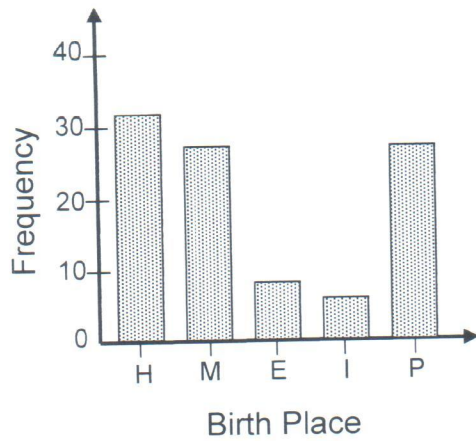
(c)



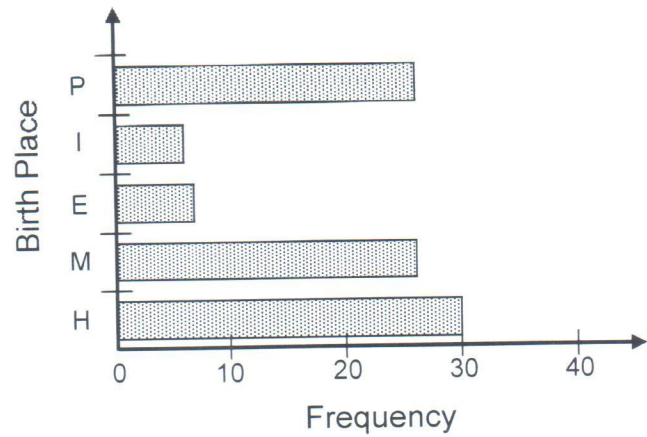
4. (a)

Birth Place	Tally marks	Frequency
H		31
M		27
E		9
I		6
P		27
Total		100

(b) (i) Column graph

DETAILS IN BIRTH PLACE

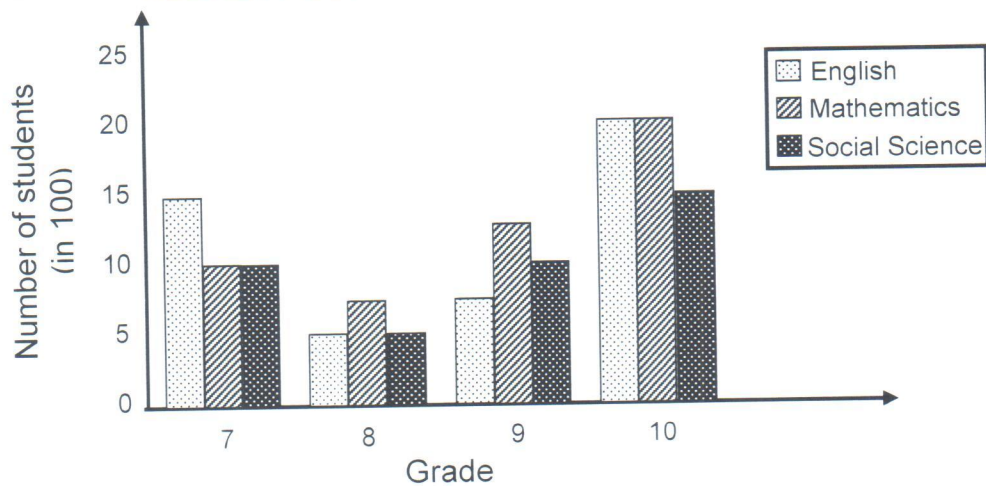
(ii) Horizontal Bar Graph

DETAILS IN BIRTH PLACE

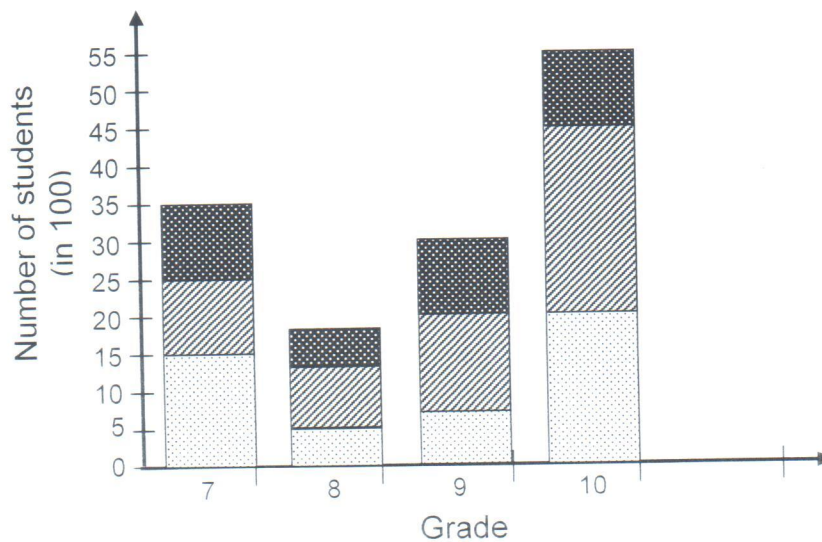
5. (a) 8000 students

(b) 2000 students

6. (a) i.

SUBJECT COMPLETIONS AT FODE DURING 2010

ii.

SUBJECT COMPLETIONS AT FODE DURING 2010

Practice Exercise 5

1. (a) Erave = 330 cm
 Madang = 360 cm
 Misima = 300 cm
 Port Moresby = 180 cm
 Rabaul = 240 cm
 Sideia = 210 cm

(b) Madang

(c) Port Moresby

(d) 120 cm

2. (a) Central and Morobe
 (b) Western highlands
 (c) 3000 telephones
 (d) 16 500 telephones

3.

Grades	Number of Females
Distinction	
Credit	  
Upper Pass	    
Pass	         
Failed	

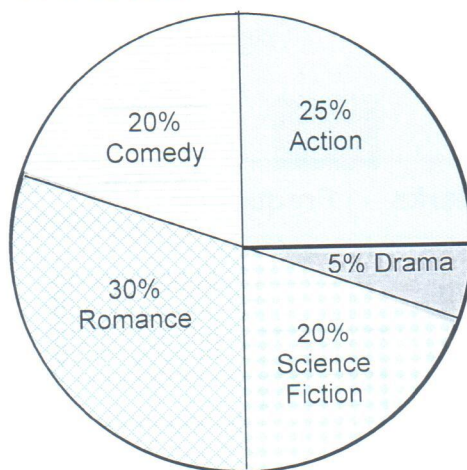
KEY:  = 5 females

4. (a) 50

(b) 120

5. (a) i. Comedy $= \frac{4}{20} \times 360^\circ = 0.2 \times 360^\circ = 72^\circ$
- ii. Action $= \frac{5}{20} \times 360^\circ = 0.25 \times 360^\circ = 90^\circ$
- iii. Romance $= \frac{6}{20} \times 360^\circ = 0.3 \times 360^\circ = 108^\circ$
- iv. Drama $= \frac{1}{20} \times 360^\circ = 0.05 \times 360^\circ = 18^\circ$
- v. Science Fiction $= \frac{4}{20} \times 360^\circ = 0.2 \times 360^\circ = 72^\circ$

(b)

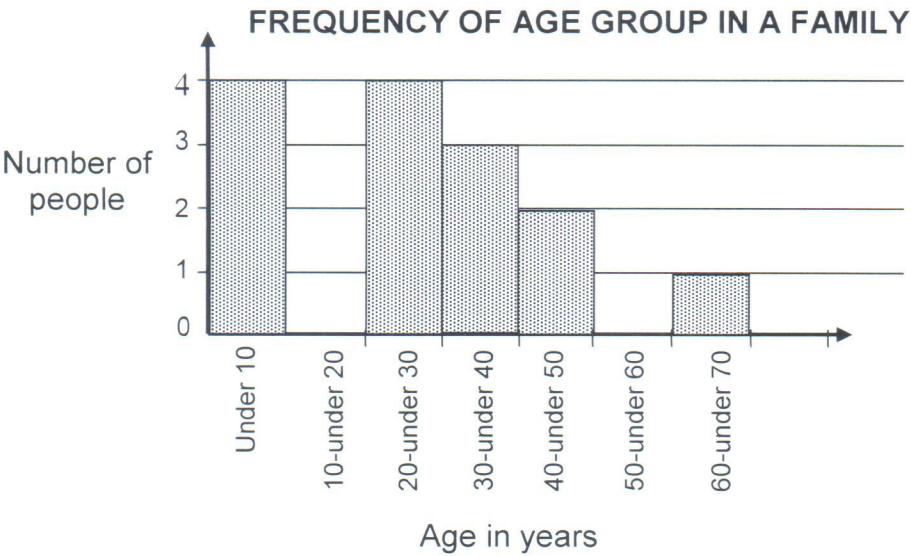
FAVORITE TYPE OF MOVIE**Practice Exercise 6**

1. (a)

FREQUENCY OF AGE GROUP IN A FAMILY

Age Group	Frequency
under 10	4
10 – under 20	0
20 – under 30	4
30 – under 40	3
40 – under 50	2
50 – under 60	0
60 – under 70	1

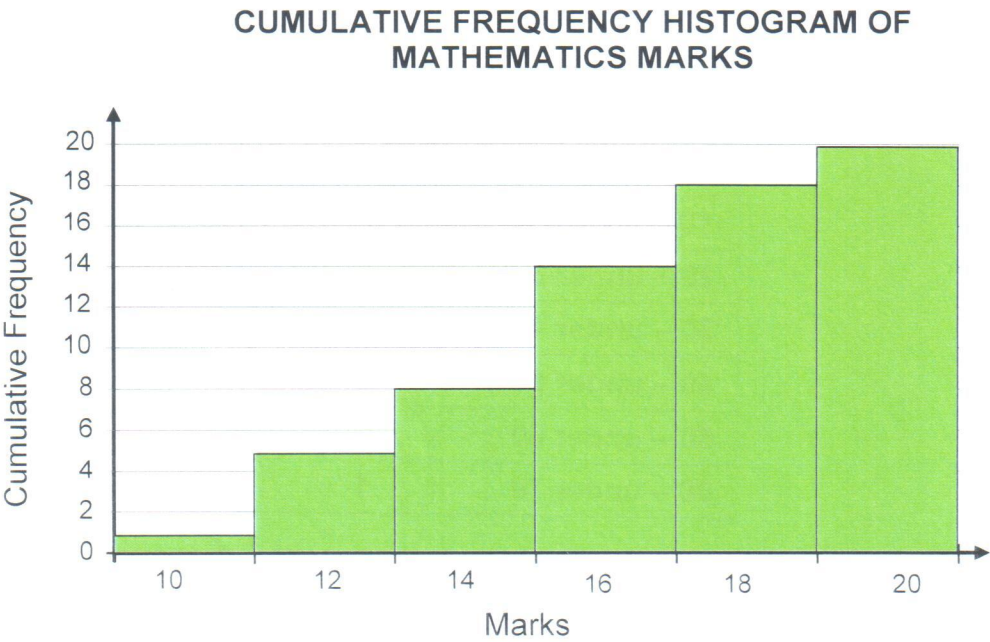
(b)



2.
(a)

Marks	Frequency	Cumulative Frequency
10	1	1
12	4	5
14	3	8
16	6	14
18	4	18
20	2	20

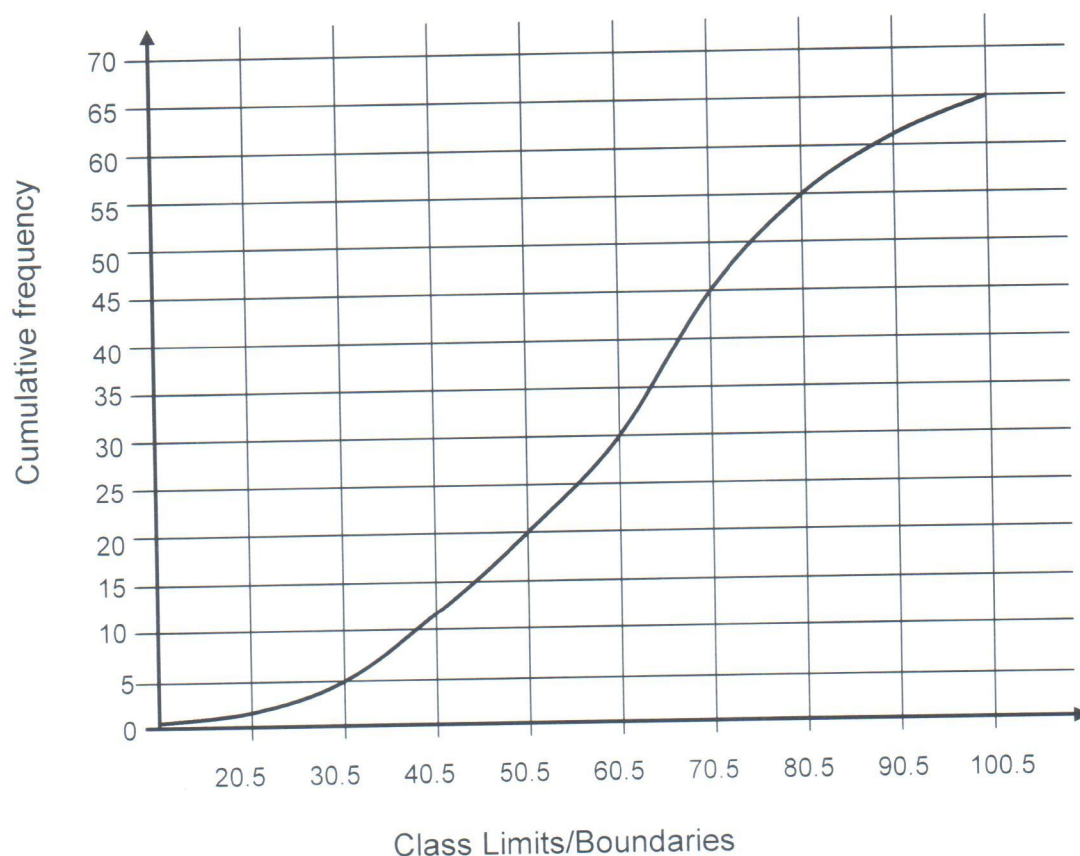
(b)



3. (a) Less than cumulative frequency table.

Heights (cm)	Frequency	Upper Class boundaries	< Cumulative frequencies
11 – 20	2	20.5	2
21 – 30	3	30.5	5
31 – 40	7	40.5	12
41 – 50	8	50.5	20
51 – 60	10	60.5	30
61 – 70	15	70.5	45
71 – 80	10	80.5	55
81 – 90	6	90.5	61
91 – 100	4	100.5	65

- (b) The Ogive



4. (a) $25 - 18 = 7$

- (b) $31 - 25 = 6$

END OF TOPIC 1

TOPIC 2

LINEAR OR STRAIGHT LINE GRAPHS

- | | |
|------------|--|
| Lesson 7: | Revision of Rate Graphs |
| Lesson 8: | Sketching Linear Graphs from Table of Values |
| Lesson 9: | The Graph of a Straight Line |
| Lesson 10: | The Slope of a Straight Line |
| Lesson 11: | Finding the Equation of a Straight Line |
| Lesson 12: | More Sketches of Straight Lines |

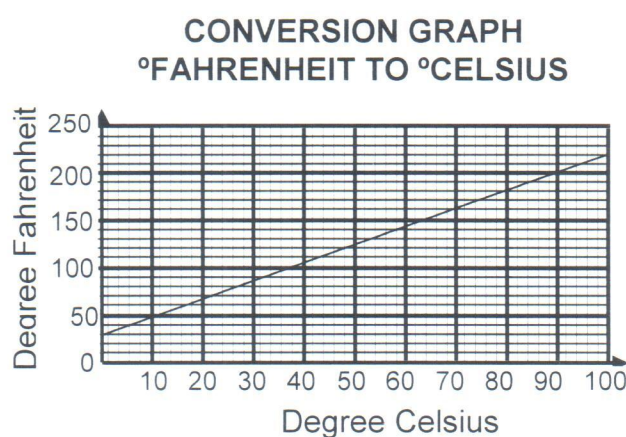
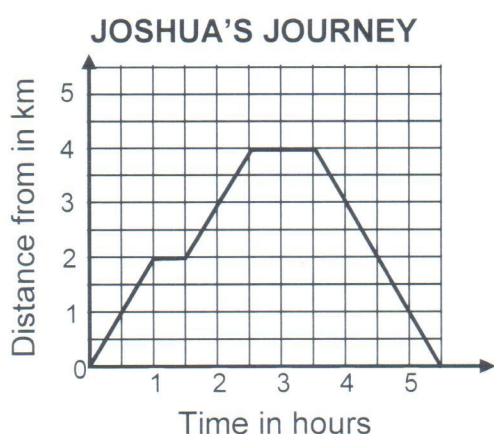
TOPIC 2: LINEAR OR STRAIGHT LINE GRAPHS

Introduction

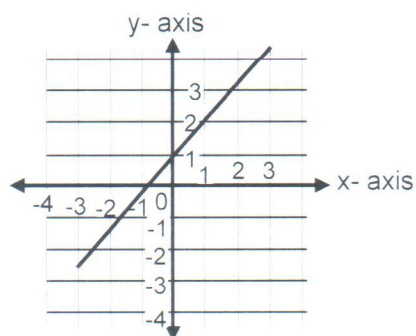


Straight Line Graphs play an important part in our modern lives. They are used extensively in Sales and Marketing, Economics, Business, Psychology, Science and Medicine.

In this topic, you will study a number of situations described and shown graphically by straight line graphs. These include rate graphs, conversion graphs and travel graphs. An example of travel graphs is shown below.



You will also study straight line graph which has been drawn on the coordinate axes and find the equation of a straight line. Below is an example of a straight line graph drawn on a coordinate axes.



You will learn how to find the equations of straight-line graphs similar to the one shown above and then study how the equation of a straight line is used in interpreting, sketching and drawing the corresponding straight line graph.

You will also extend your knowledge of travel graphs. The relationship between the slope and rate of change of a travel graph is introduced and developed, including the gradient–speed relationship.

Lesson 7: Revision of Rate Graphs



Welcome to the first lesson of your Grade 10 Unit 4 Topic 2.



In this lesson, you will:

- revise the meaning of rates
- find the rates from rate graphs.

Let us start the lesson by looking at the following statements:

1. If 8 kg of rice cost K39.20.

$$\text{Then the cost of one kg} = \frac{\text{K}39.20}{8} = \text{K}4.90$$

i.e. the cost **per** kg = K4.90

2. If the cost of one dozen eggs = K10.80

$$\text{Then the cost of one egg} = \frac{\text{K}10.80}{12} = 90\text{t}$$

i. e. the cost **per** egg = 90t

3. A car travels 60 km in a time of $\frac{1}{2}$ hour.

$$\text{The distance travelled in one hour} = \frac{60}{0.5} = 120 \text{ km}$$

i.e. distance travelled **per** hour = 120 km

4. The population of a village increases by 20 people over 5 years.

$$\text{The average increase in one year is} = \frac{20}{5} = 4 \text{ people}$$

i.e. the average increase **per** year = 4 people

Notice that in each case we have worked out the amount for one unit of the quantity.
i.e. the amount per unit of the quantity.

Therefore, in (1) above, the price per unit (per kg) of rice = K4.90 and in (3) above, the distance travelled per unit (per hour) = 120 km.

The amount for one unit of a quantity is called the **rate**. In other words:

Rate means for **every single item** or **unit**.

For example:

- a) The rate of eggs = 90¢ per egg.
- b) The rate of distance travelled = 120 km per hour.
- c) The rate of population increase = 4 people per year...etc.

In the previous examples, we have found the rate by using arithmetic. We can also find a rate by carefully drawing a graph showing the given information. We call this graph a **rate graph**. From the graph, we are able to find the rate per unit of the quantity. Other information can also be found from the graph.

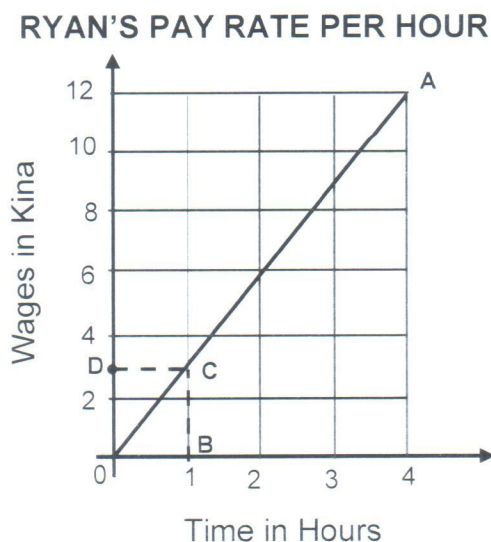
Here is an example of finding a rate from a graph.

Ryan works 4 hours and is paid K12. What is his rate of pay?

To solve this problem, we first label time (hours) on the horizontal of a grid and money (Kina) on the vertical axis of the grid.

Now, since 4 hours corresponds to K12, we plot the point **A** (4, 12) corresponding to this pair of numbers on the grid. We then join this point with a straight line to the origin **O**. What we are saying is that: For zero hours = 0 Kina which is the other point (0, 0) or the origin.

To find the rate of pay per hour, we next draw a vertical line from a point **B** on the horizontal axis where time equals 1 hour up to meet the line **OA** at the point **C**. From **C**, we draw a horizontal line to meet the vertical axis at **D**. We then read off the amount of money at **D**. This amount will give us the rate of pay, which is actually the amount earned in one hour.



From this graph, Ryan's rate of pay per hour is K3.

Now, suppose we want to convert a quantity from one set of unit to another. For example, converting PNG Kina to Australian dollars, or distance in kilometres to the corresponding distance in miles. We can do this by using a similar graphical method

like the one above, where we found the rate for an item first. The type of graph we draw is called a **conversion graph**.

Note:

It is only possible to convert units, which measure the same type of quantity. We could not convert Kina (money) to Kilometres (distance) since these units measure different items.

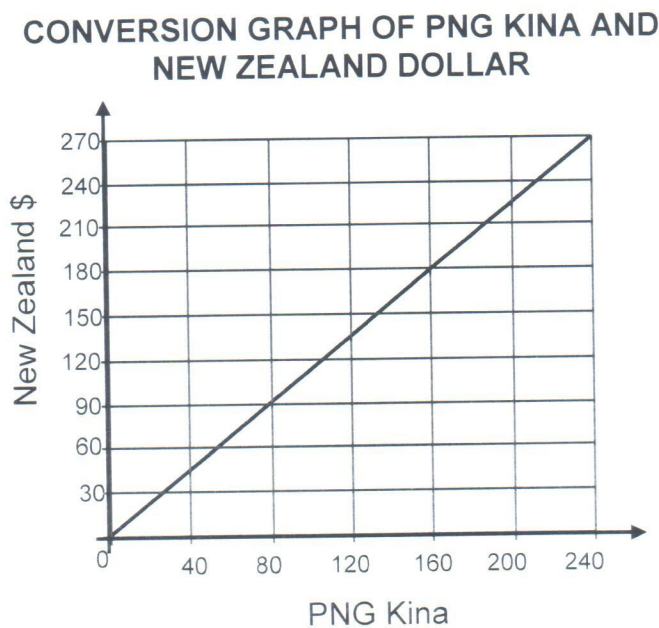
Here is an example of drawing a conversion graph.

Betty wanted to exchange her PNG K240 to New Zealand Dollars. The Bank told her, she would get on conversion NZ\$270 for her K240. Draw a conversion graph between PNG Kina and NZ\$ and find the rate. In other words, find out how much is PNG K1 worth in New Zealand Dollars (NZ\$).

Below are the steps involved in drawing the conversion graph.

- (i) We first label the axes of a grid. We will choose the horizontal axis of the grid to be Kina (up to K240) and the vertical axis to be in NZ\$ (up to NZ\$270). So, one spot on the graph is already found (K240, NZ\$270). This means: $K240 = NZ\$270$
- ii) Mark on the graph the point with co-ordinates (240, 270) from (i) above
- (iii) The other point on the graph is the Origin (0, 0). Draw a straight line (the conversion line) from (0,0) to the point we have just plotted (240, 270).

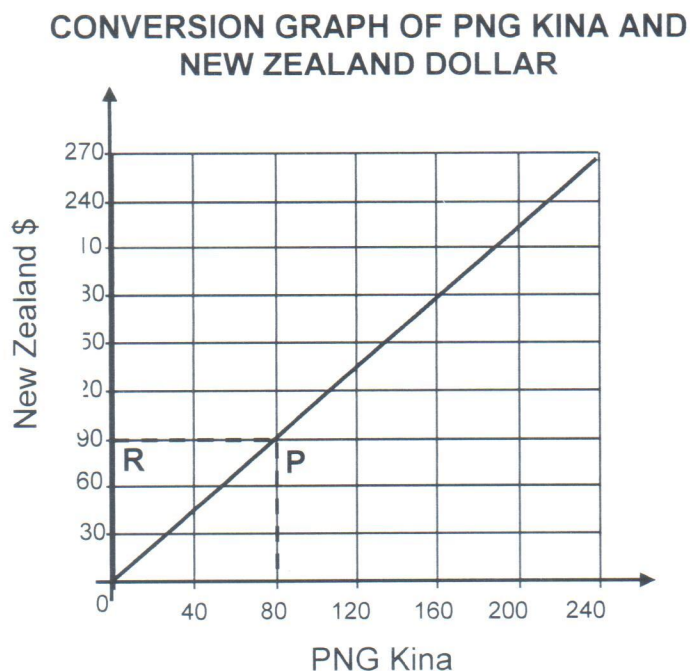
The Conversion Graph would look like this.



We can now use our conversion graph to find how many New Zealand dollars is equivalent to K120.

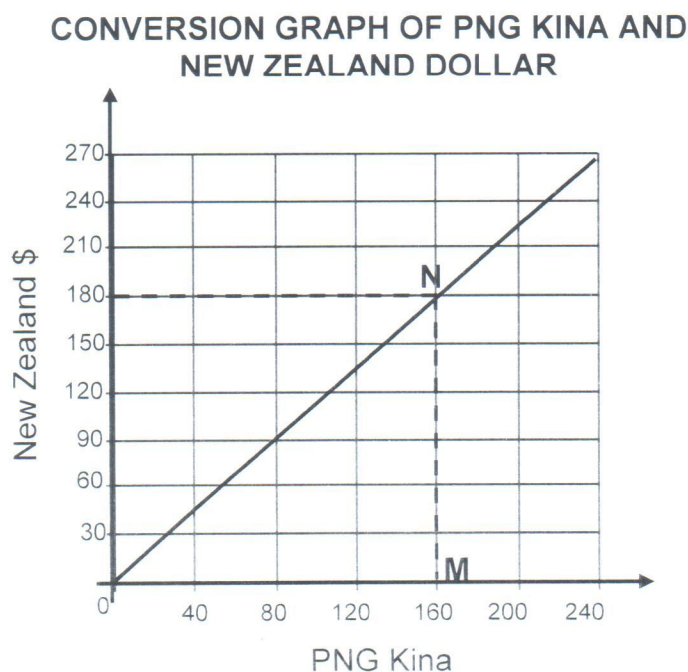
To do this, we draw a vertical line or look up from K80 on the horizontal axis to meet the conversion line at **P**, and from **P**, draw a horizontal line or look across to the vertical axis to point **R** and read off the number of NZ\$ at this point. You will see that $K80 = \text{NZ\$}90$. (See Figure 1 below).

Figure 1



To find, how much is NZ \$180 equivalent to in Kina, we draw a horizontal line or look from the vertical axis at the point NZ \$180 across to where it meets the conversion line (point **N**). From **N**, draw a vertical line or look downwards to meet the horizontal axis at **M** (see Figure 2 below). You will see that $\text{NZ \$}180 = K160$.

Figure 2



Conversion graphs are particularly useful when we have to convert so many items. For example: A Bank Teller might have to convert different quantities of one currency into another many times a day. This would be difficult to do if a calculator was not at hand.

Similarly, if a teacher sets a class test marked out of 60 and wanted to convert the marks to percentages (that is out of 100), he would find it very useful to draw a conversion graph which would enable him to convert his students' marks out of 60 to percentage marks very quickly.

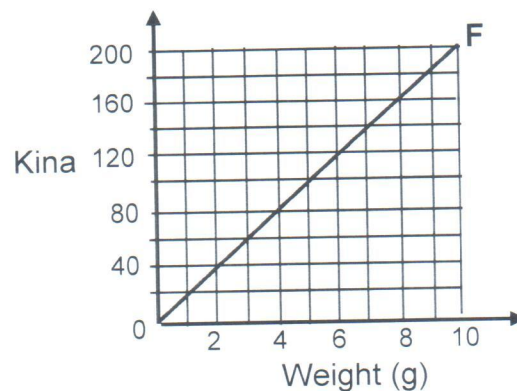
Examples

1. Given that 10g of gold cost K200:

- (a) Draw a graph to find the rate for the gold, i.e. how much per gram of gold.

First draw the horizontal and vertical axes and label them Kina and Weight respectively

After marking the point corresponding to 10 g for K200, i.e. **F** (10, 200), join it to the Origin (0, 0) with a straight line. To find the gold rate, read off the value on the vertical axis corresponding to 1 g on the horizontal axis, you see that it is K20.



- (b) What is the cost of 6 g of gold?

Draw a line from the horizontal axis where the weight is 6 g, look up to the slanting line. From where the two lines meet, draw a line or look across to the vertical axis. The value on this axis at this point will give us the cost, which is K120 in this case.

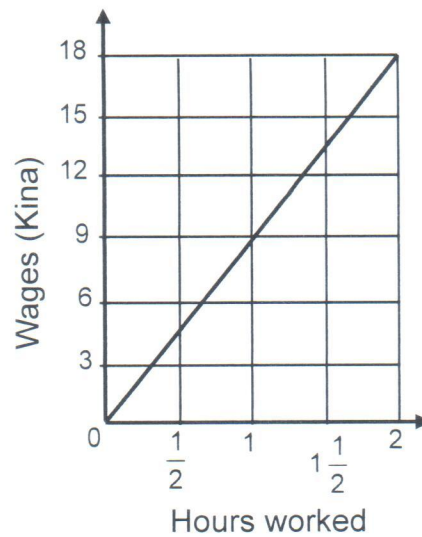
Answer: K120

- (c) How many grams of gold can be bought for K180?

Draw a horizontal line or look from the point on the vertical axis where we have K180. From where this line meets the slanting line, draw a vertical line or look down to the horizontal axis. Where this line meets the axis will be the weight of gold, which in this case is 9 g.

Answer: 9 g

2. The graph below shows the amount Mary earns as wages.



From the graph find Mary's rate of pay.

Rate of pay = K9/hour

Answer: K9/hour

NOW DO PRACTICE EXERCISE 7

**Practice Exercise 7**

1. (a) If 10 litres of petrol cost K42, what is the rate (cost) per litre?

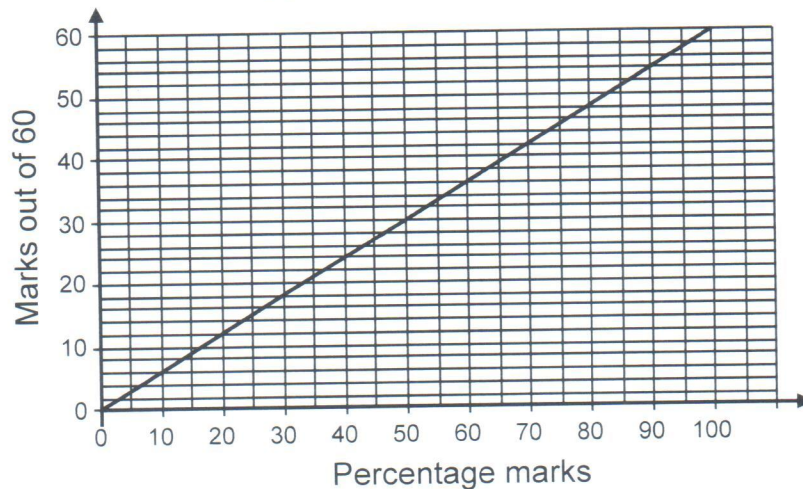
Answer: _____

- (b) The population of a town increases by 9000 people in 3 years. What is the rate of increase per year?

Answer: _____

2. The graph given below converts a mark out of 60 to a percentage mark.

GRAPH FOR CONVERTING MARKS OUT OF 60 TO PERCENTAGE MARKS



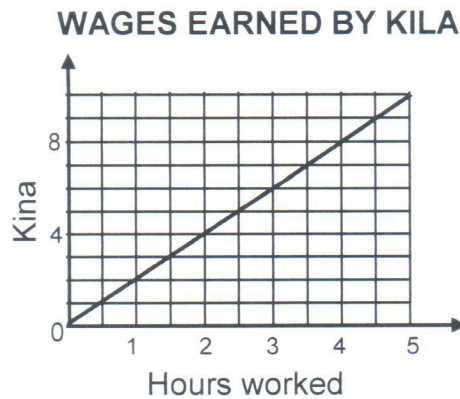
- (a) Kira scored 36 marks out of 60 in a Science test. What was her percentage mark?

Answer: _____

- (b) Robin was awarded 90% in the same Science test. How many marks did he originally score out of 60?

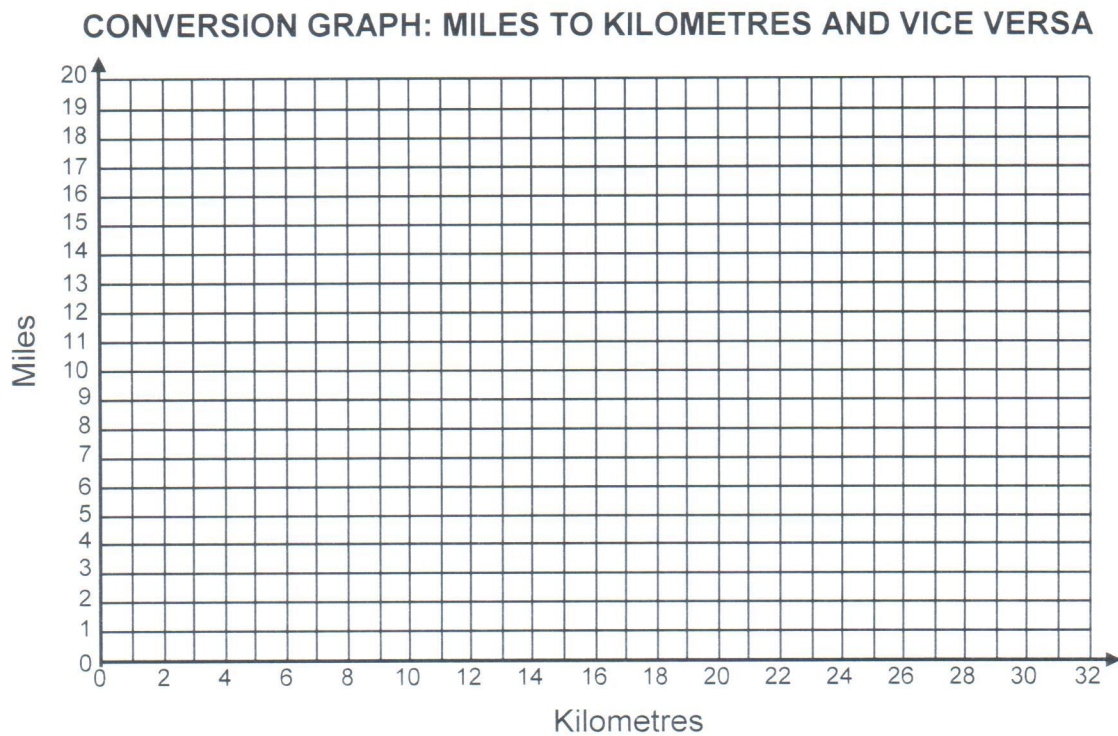
Answer: _____

3. Study this graph showing the wages earned by Kila and then find his rate of pay.



Answer: _____

4. (a) Using the fact that 20 miles = 32 kilometres, draw a conversion line for these two quantities on the grid below.



- (b) Using the graph to convert:

(i) 20 kilometres into miles

Answer: _____

(ii) 10 miles into kilometres

Answer: _____

(iii) A race is 14 miles long. What is the length of the race to the nearest kilometre?

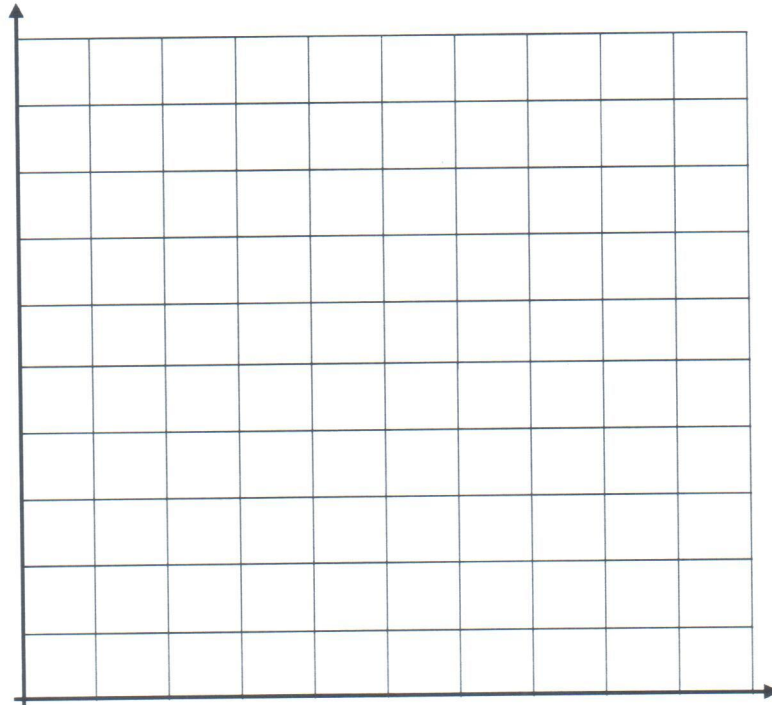
Answer: _____

5. A shop sells eggs for K8.00 per dozen.

(a) Complete the table:

Dozens of eggs	0	1	2	3	4	5
Kina	0	8				

- (b) Use the information in the table to draw a Ready Reckoner. Put Kina on the vertical axis and Dozens of eggs on the horizontal axis.



- (c) Use your graph to find the cost of:

(i) 3 dozens of eggs

Answer: _____

(ii) 4.5 dozens of eggs

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 8: Sketching Linear Graphs from Table of Values



You have revised rates and discovered rates from graphs in the previous lesson.



In this lesson, you will:

- define and create a table of values
- sketch linear graphs using table of values.

There are several other methods of graphing linear graphs. Sometimes graphing lines using an equation involves the same methods as using a table of values. Since we graph lines in the coordinate plane, it is necessary to understand how to connect graphs, tables and equations.

The equation of a line expresses a relationship between x and y values on the coordinate plane. For instance, the equation $y = x$ expresses a relationship where every x value has a corresponding y value. The equation $y = 2x$ expresses a relationship in which every y value is double the x value, and $y = x + 1$ expresses a relationship in which every y value is 1 greater than the x value.

Since every equation is a relationship of x and y values, we can create a table of values for any line. These are just the x and y values that are true for the given line. In other words, a table of values is simply some of the points that are on the line.

You can graph any equation using a table of values.

A table of values is a graphic organizer or chart that helps to determine two or more points that can be used to create a graph.

Below is an example of table of values.

x	$2x + 1$	y	Ordered Pairs
-2	$2(-2) + 1$	-3	$(-2, -3)$
0	$2(0) + 1$	1	$(0, 1)$
2	$2(2) + 1$	5	$(2, 5)$

In the table, there are 4 columns as described below:

- The **first** column is for the **x coordinate**. For this column, you can choose any number you wish. Try to choose numbers that can be graphed on your graph. For example, if your x -axis only extends to 10, don't choose 12 as an x -coordinate.
- The **second** column is for **substituting** x into the equation in order to solve for y . So, whatever value you chose for x , you will substitute back into the equation and solve to find the y value.

- The **third** column is for the **y** value. After substituting your x value into the equation, your answer is the y-coordinate.
- The **last** column is for your **ordered pair**. Your ordered pair is the x value and the y value. This is the point on your graph.

In order to graph a line, you must have two points. For any given linear equation, there are an infinite number of solutions or points on that line.

If you find two of the solutions, then you can plot the two points and draw a line through them. This will be the line that represents the equation. Every point on that line is a solution to the equation.

Now let us look at some examples.

Example 1

Graph the line for the equation $y = 2x + 1$.

Solution:

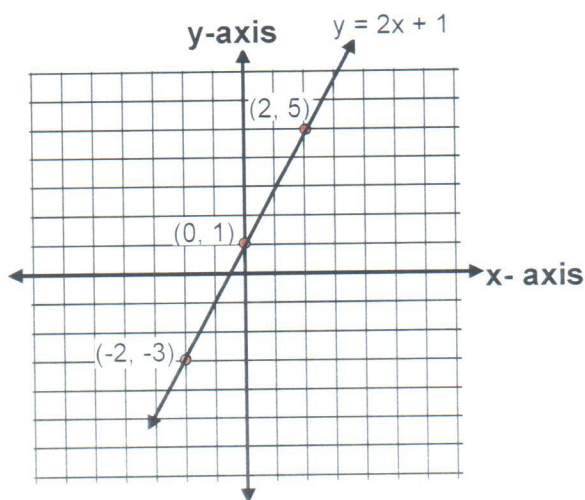
Create the table of values by assigning values for x to find the values for y.

x	$2x + 1$	y	Ordered Pairs
-2	$2(-2) + 1$	-3	$(-2, -3)$
0	$2(0) + 1$	1	$(0, 1)$
2	$2(2) + 1$	5	$(2, 5)$

In the table, we choose -2, 0, and 2 as values for x. After substituting the three values into the equation $y = 2x + 1$, we found the values for y to be: -3, 1, and 5.

Hence the ordered pairs that we found were: $(-2, -3)$, $(0, 1)$, and $(2, 5)$

Now we plot these points on the coordinate axis. Then using a ruler, draw a line through these points.



This is the line for the equation $y = 2x + 1$. This line goes on forever, so there are infinite solutions to the equation.

Example 2

Graph the line for the equation $y = -\frac{1}{2}x - 1$.

Solution: This equation $y = -\frac{1}{2}x - 1$, has a fraction as the coefficient of x . This becomes a little trickier in choosing x coordinates because we could end up with a fraction for the y coordinate. This then becomes a little more difficult to graph. Hence, the easier way in assigning values for x , choose 0 or a multiple of the denominator as your x -coordinates.

Create the table of values.

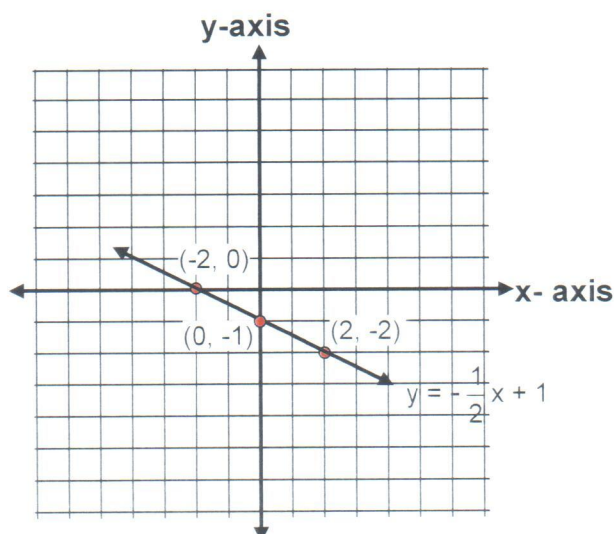
Table of values

x	$-\frac{1}{2}x - 1$	y	Ordered Pairs
-2	$-\frac{1}{2}(-2) - 1$	0	(-2, 0)
0	$-\frac{1}{2}(0) - 1$	-1	(0, -1)
2	$-\frac{1}{2}(2) - 1$	-2	(2, -2)

In the table, we choose -2, 0, and 2 as values for x . After substituting the three values into the equation $y = -\frac{1}{2}x - 1$, we found the values for y to be: 0, -1, and -2.

Hence the ordered pairs that we found were: (-2, 0), (0, -1), and (2, -2)

Now we plot these points on the coordinate axis. Then using a ruler, draw a line through these points.



Note:

If the coefficient of x in the linear equation is a fraction, choose zero (0) or a multiple of the denominator as your x -coordinates.

Now let us have another example.

Example 3

Find five solutions for the equation $y = 2x - 3$. Then graph the equation.

Solution: Create the table of values.

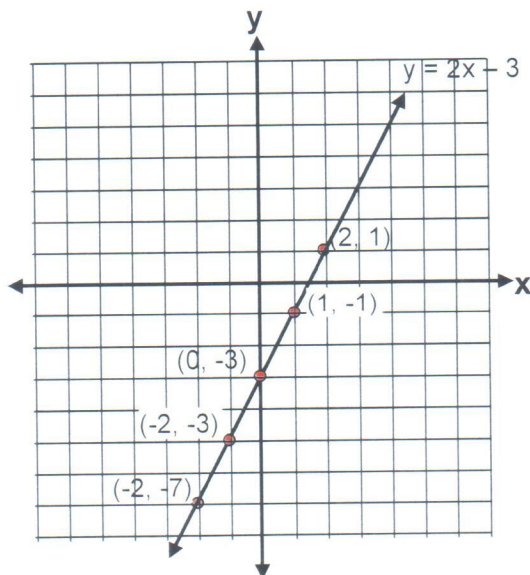
Table of values

x	$2x - 3$	y	Ordered Pairs
-2	$2(-2) - 3$	-7	$(-2, -7)$
-1	$2(-1) - 3$	-5	$(-1, -5)$
0	$2(0) - 3$	-3	$(0, -3)$
1	$2(1) - 3$	-1	$(1, -1)$
2	$2(2) - 3$	1	$(2, 1)$

In the table, we choose -2, -1, 0, 1, and 2 as values for x . After substituting the five values into the equation $y = 2x - 3$, we found the values for y to be: -7, -5, -3, -1 and 1. Hence, the ordered pairs that we found on the graph were:

$(-2, -7)$, $(-1, -5)$, $(0, -3)$, $(1, -1)$, and $(2, 1)$

Now we plot these points on the coordinate axis. Then using a ruler, draw a line through these points.



NOW DO PRACTICE EXERCISE 8

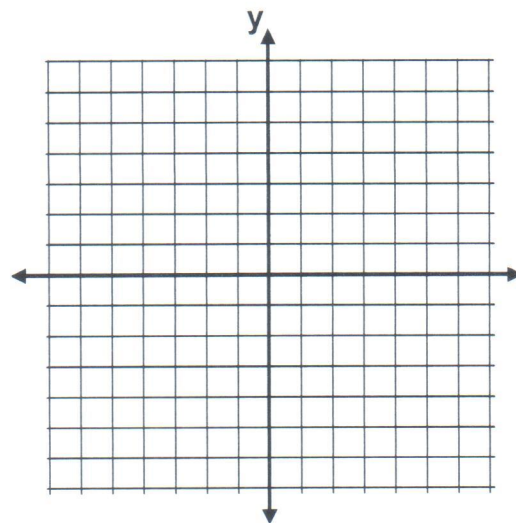


Practice Exercise 8

1. Complete the following table of values at the right using each of the following equation. Then graph the equation on the grid at the right.

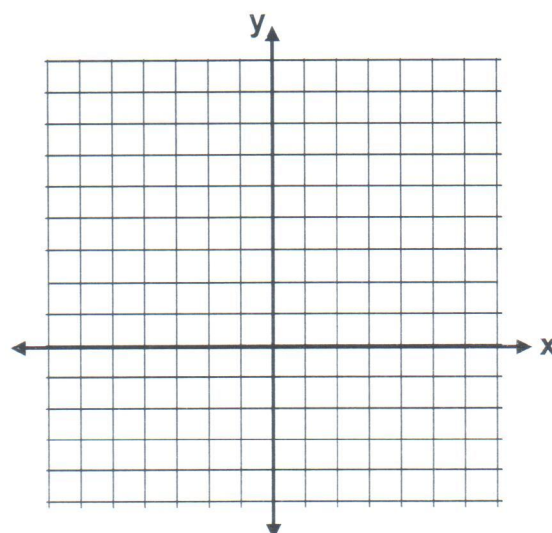
a. $y = -\frac{5}{3}x - 2$

x	$-\frac{5}{3}x - 2$	y	Ordered pairs
-3			
0			
3			



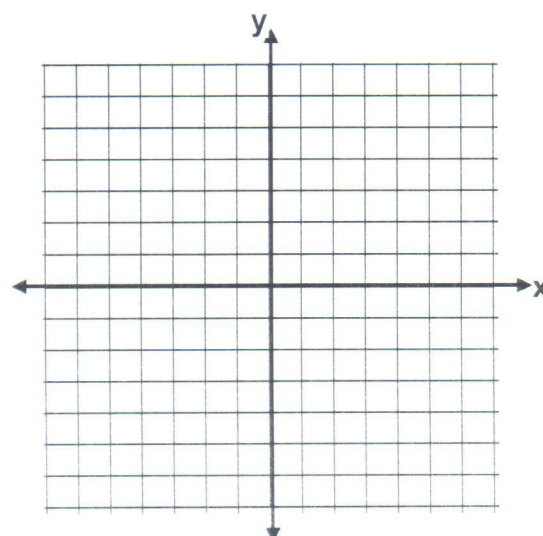
b. $y = 3x + 2$

x	$3x + 2$	y	Ordered pairs
-2			
-1			
0			
1			
2			



c. $y = -x + 2$

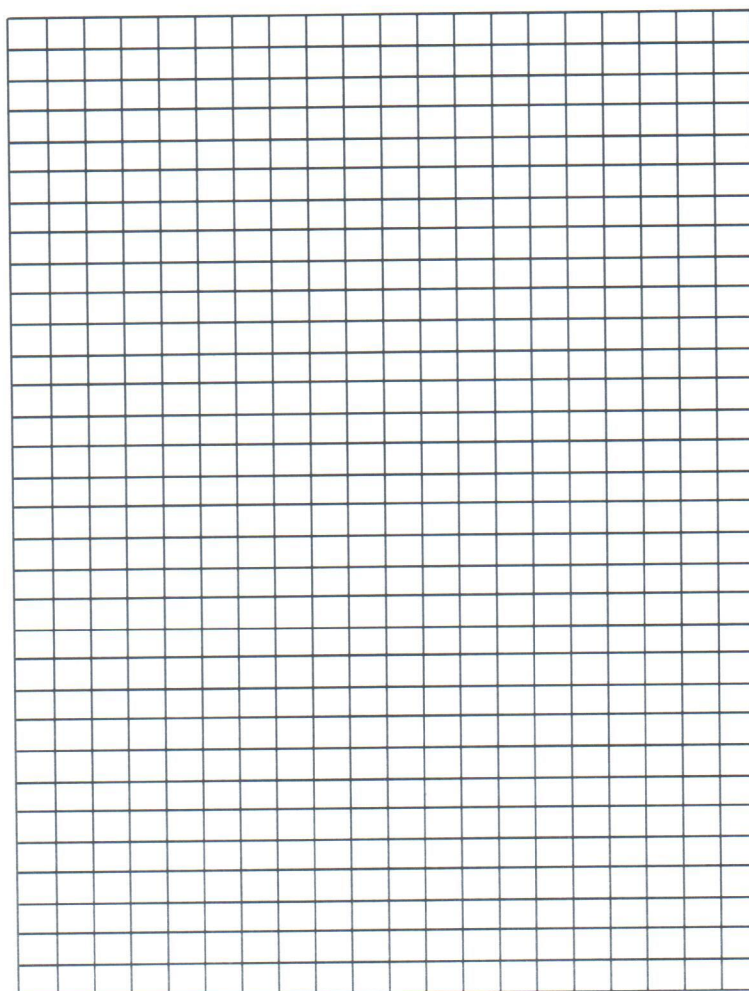
x	$-x + 2$	y	Ordered pairs
-2			
-1			
0			
1			
2			



2. Given the equation $y = 7 - 5x$.

- a. Create a table of values by assigning the following values for x : -1, 0, 1, 2, and 3 to find the values for y .

- b. Sketch the graph on the grid below.



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: The Graph of a Straight Line



You learnt to create table of values and use them to graph linear equations in the previous lesson.



In this lesson, you will:

- identify the x- and y-intercepts of a straight line
- calculate the x – and y-intercepts of a straight line
- draw graphs of straight lines using a given equation.

Often we need to know the general shape and location of the graph. In such cases, a sketch graph is drawn instead of plotting a number of points to obtain the graph.

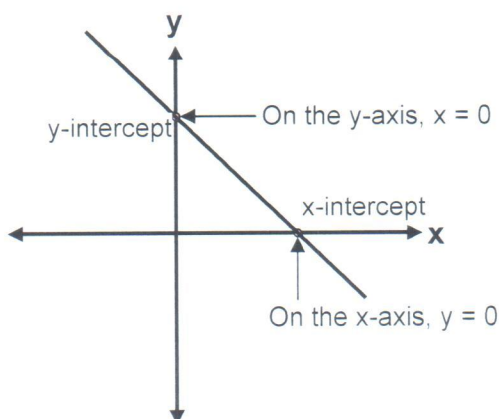
There are other methods for graphing straight lines, such as graphing from the intercepts or graphing from the y-intercept and the slope.

There are several ways to graph straight line given its equations.

Two points are needed to obtain a straight line graph. It is simpler to find points of intersection of the graph with the axes. These points are called the **x-** and **y-intercepts**.

The **x-intercept** is the point where the line crosses the x-axis.
The **y-intercept** is the point where the line crosses the y-axis.

The x- and y- intercepts of a graph



How to find the intercepts?

The y- coordinate of any point on the x-axis is 0. Therefore, to find the x-intercept, we set $y = 0$ in the given equation and solve for x .

The x-coordinate of any point on the y-axis is 0. Therefore, to find the y-intercept, we set $x = 0$ in the given equation and solve for y .

Example 1

Find the x-intercepts of $y = 2x + 10$.

Solution: To find the x-intercept, set $y = 0$.

$$y = 2x + 10$$

$$0 = 2x + 10$$

$$2x = -10$$

$$x = -5$$

The x-intercept is -5.

Example 2

Calculate the x-intercept in the equation $y = 3x - 12$.

Solution: To find the x-intercept, set $y = 0$.

$$y = 3x - 12$$

$$0 = 3x - 12$$

$$-3x = -12$$

$$x = 4$$

The x-intercept is 4.

To find the y-intercepts of a graph, we must find the value of y when $x = 0$ because every point on the y-axis, $x = 0$.

Drawing the graph

It was mentioned earlier that two points determine a straight line. And the two most important points are the x- and y-intercepts. Therefore, whenever we draw the graph of a straight line we always plot these two points.

Example 3

Find and plot the x- and y- intercepts, and draw the graph of $y = 2x + 6$.

Solution: The x- intercept: $y = 2x + 6$

$$0 = 2x + 6$$

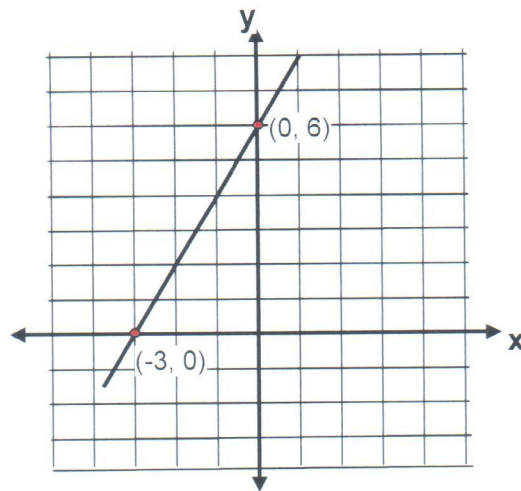
$$-2x = 6$$

$$x = -3$$

Therefore the x intercepts is $(-3, 0)$.

The y-intercept is the constant term 6 or $(0, 6)$.

We mark these two on the coordinate axis.



The graph of the equation is the straight line that passes through these intercepts.

Example 4

Find the x- and y-intercepts, and draw the graph of $5x - 2y = 10$.

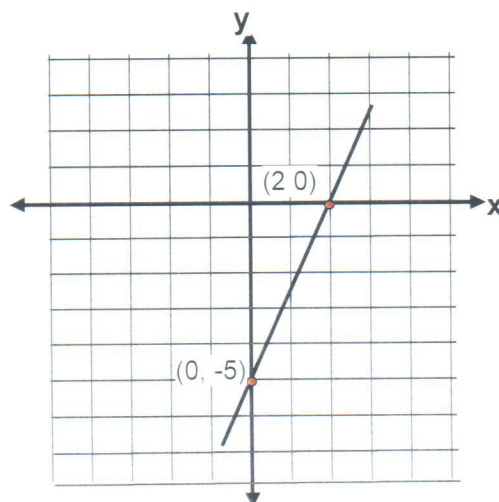
Solution: Find the x-intercept: Set $y = 0$.

$$\begin{aligned}5x - 2y &= 10 \\5x - 2(0) &= 10 \\5x &= 10 \\x &= 2\end{aligned}$$

Find the y-intercept: Set $x = 0$.

$$\begin{aligned}5(0) - 2y &= 10 \\-2y &= 10 \\y &= -5\end{aligned}$$

Here is the graph.



Example 4

Sketch the graph of $y = \frac{2}{3}x - 4$.

Solution: $y = \frac{2}{3}x - 4$

Find the x- intercept: when $y = 0$; $0 = \frac{2}{3}x - 4$

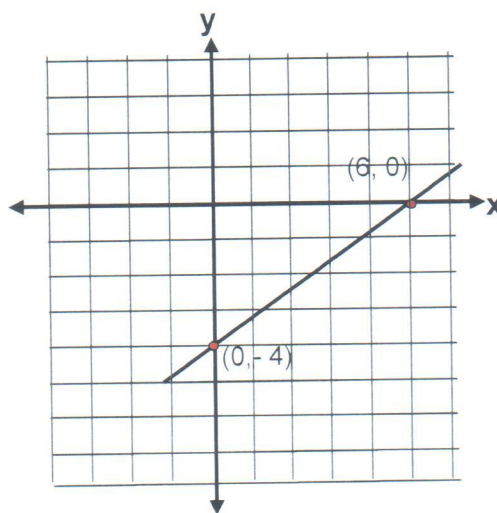
$$\frac{2}{3}x = 4$$

$$2x = 12$$

$$x = 6$$

Find the y-intercept: when $x = 0$; $y = \frac{2}{3}(0) - 4$
 $y = -4$

Sketching the graph on the coordinate axis, the graph is shown below.



Example 5

Draw the graph of $y = 2x$.

Solution: Find the x-and y- intercepts.

The x-intercept, set $y = 0$.

$$y = 2x$$

$$0 = 2x$$

$$x = 0$$

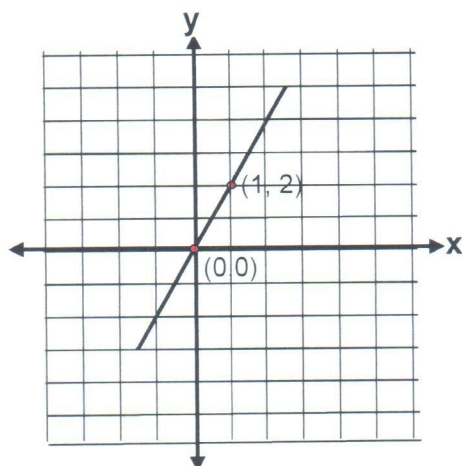
The y-intercept is 0.

The intercepts coincide at $(0, 0)$ hence, the graph passes through the origin.

To draw the graph, we need to find another point by choosing a value for x to find a corresponding value for y .

So, if $x = 1$, then $y = 2(1) = 2$. The point $(1, 2)$ is on the graph.

Here is the graph.



Example 6

Sketch the graph of $y = \frac{2}{3}x$.

Solution: Find the x- and y- intercepts.

The x-intercept, set $y = 0$.

$$y = 2x$$

$$0 = 2x$$

$$x = 0$$

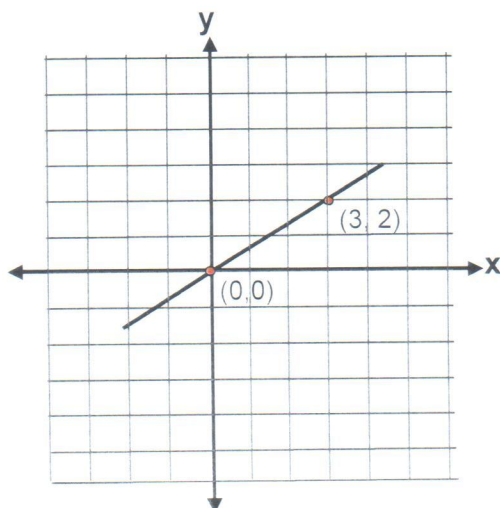
The y-intercept is 0.

Again, the graph passes through the origin. To find another point, choose an integer for x in such a way that y also will be an integer. Choose x equal to the denominator 3.

$$\text{Hence, if } x = 3, \text{ then, } y = \frac{2}{3}x = \frac{2}{3}(3) = 2$$

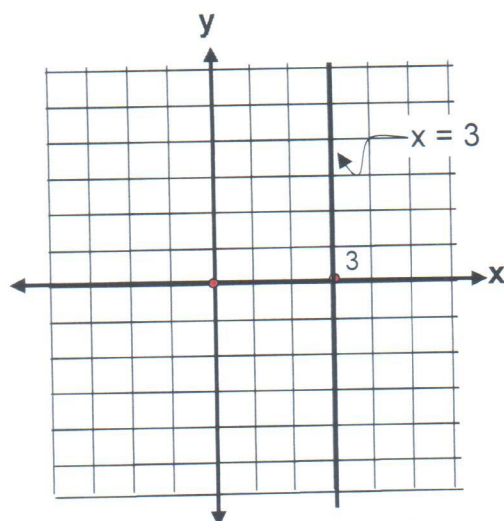
Therefore the point $(3, 2)$ is on the graph.

Here is the graph.



Example 7

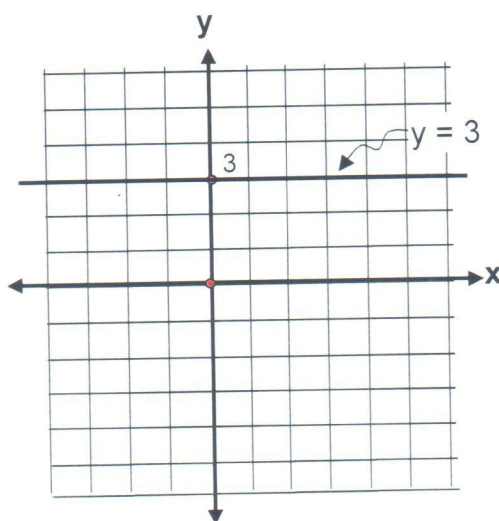
Graph the equation $x = 3$.



$x = 3$ is the same as $0y + x = 3$. For any values of y , x is 3. The graph is a **vertical line**.

Example 8

Graph the equation $y = 3$.



$y = 3$ is the same as $y + 0x = 3$. For any values of x , y is 3. The graph is a **horizontal line**.

NOW DO PRACTICE EXERCISE 9

**PRACTICE EXERCISE 9**

1. Find the x- and y-intercepts of each of the following equation.

a. $y = 2x + 4$

Answer: _____

b. $y = 3x - 12$

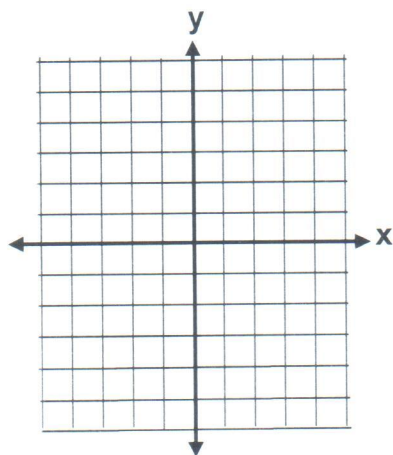
Answer: _____

c. $4x - y + 1 = 0$

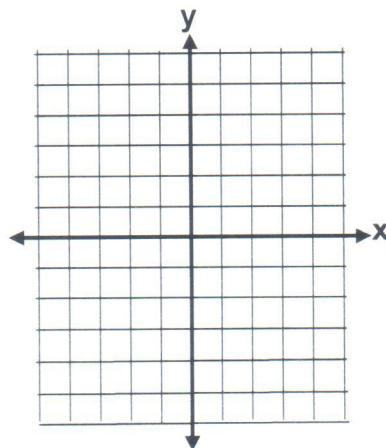
Answer: _____

2. Mark the x- and y-intercepts, and draw the graph of each of the following equation.

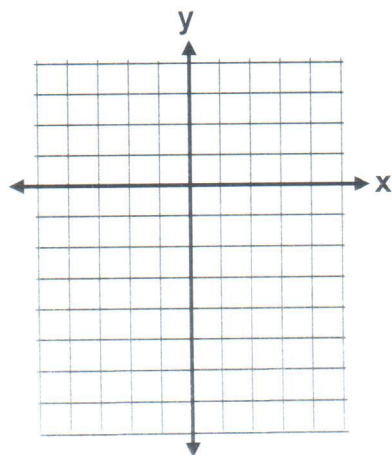
a. $x - y = 3$



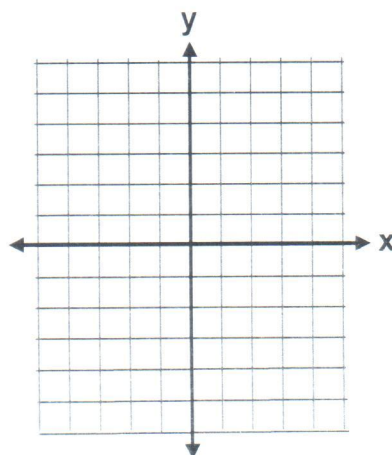
b. $y = 4x + 2$



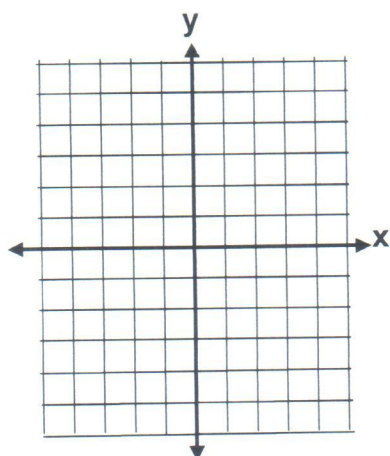
c. $y = 2x - 6$



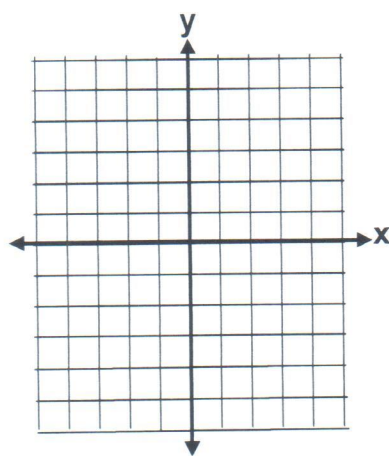
d. $y = -3x + 3$



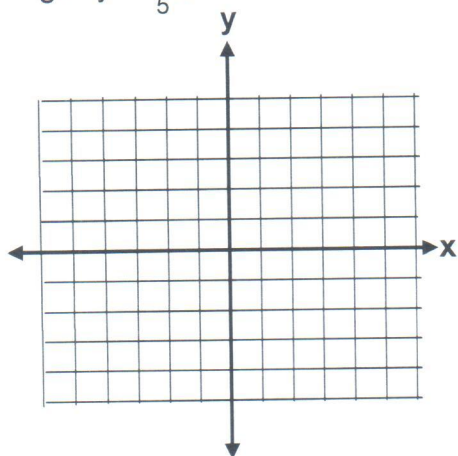
e. $2x - 3y - 6 = 0$



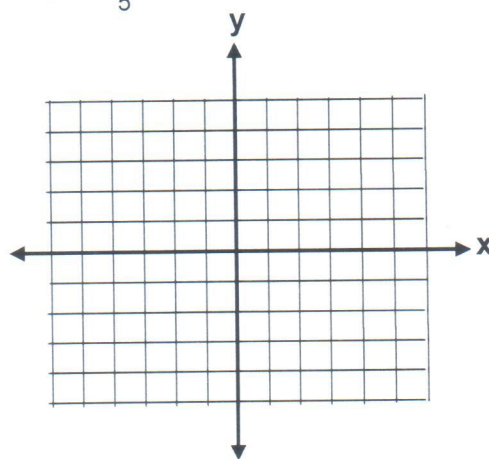
f. $x - 2y + 2 = 0$



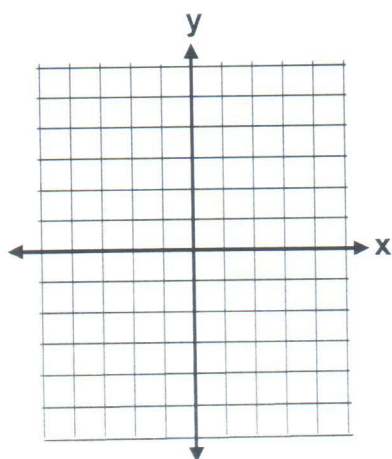
g. $y = \frac{3}{5}x$



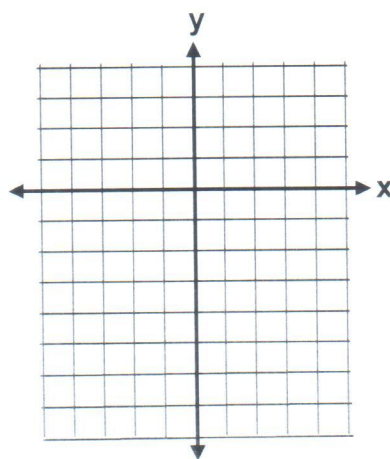
h. $y = -\frac{2}{5}x$



i. $x = -4$



j. $y = -6$



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 10: The Slope of a Straight Line



In the previous lesson you learnt the equation and graph of a straight line. You found the intercepts of a straight line.



In this lesson, you will;

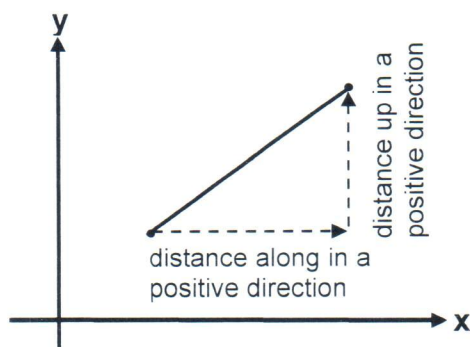
- define slope or gradient of a straight line
- calculate the gradient of a straight line.

One of the most important properties of a straight line is how it angles away from the horizontal. This concept is called the "slope" or "gradient" of the line.

In Grade 9 Unit 4 Topic 2 Lesson 7, you learnt that the gradient (also called slope) of a straight line shows **how steep** a line is.

To calculate the gradient or slope we consider two points.

Then we divide the distance **up** the graph by the distance **along** the graph.



$$\text{Thus, Gradient} = \frac{\text{Distance up}}{\text{Distance along}}$$

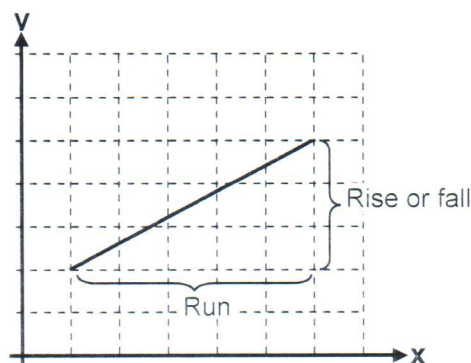
Notice that the **distance up** is the **change in y** coordinates and the **distance along** is the **change in x** coordinates.

Sometimes the horizontal change is called "**run**", and the vertical change is called "**rise**" or "**fall**":

Thus we can also say that;

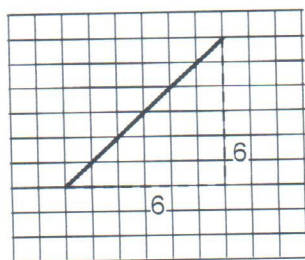
$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x}$$

$$\text{Or Gradient} = \frac{\text{Rise}}{\text{Run}}$$



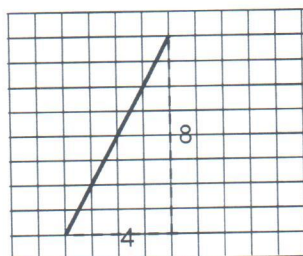
They are just different words, none of the calculations change.

Examples



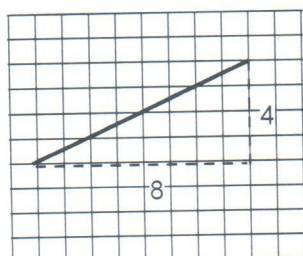
$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{6} = 1$$

So the gradient is equal to 1.



$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{8}{4} = 2$$

The line is steeper, and so the gradient is larger.



$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{2}{8} = \frac{1}{4}$$

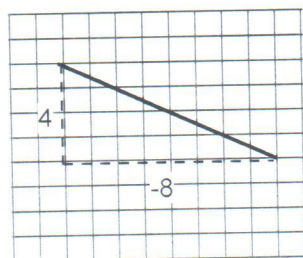
The line is less steep, and so the gradient is smaller.

The slope or gradient of a straight line can be positive, negative, zero or undefined.

The slope of a line is **positive** if the line **leans to the right** or **goes upward from left to right**.

All the examples above have positive slopes.

The slope of a line is **negative** if the line **leans to the left** or **goes downward from right to left**.

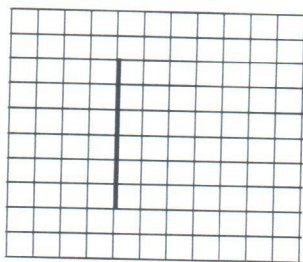


$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{4}{-8} = -\frac{1}{2}$$

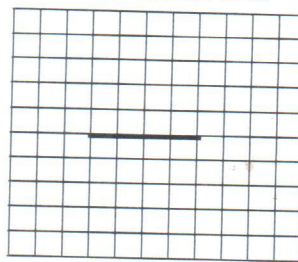
The slope of a line is **zero**, if the line is horizontal. This is because even though the value of **x** changes, the value of **y** does not.

The slope of a line is **undefined**, if the line is vertical. This means that a vertical line has no slope.

Vertical line



Horizontal line

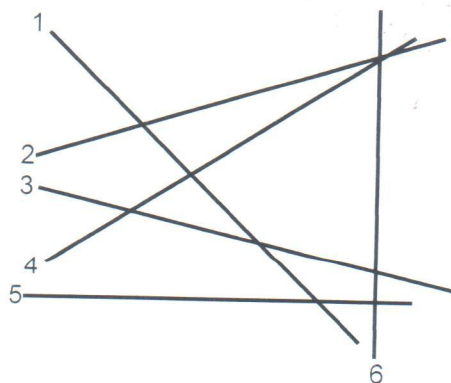


$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{0} = \text{undefined} \quad \text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{0}{4} = 0$$

In summary, the gradient of a straight line can be described in this way:

- A line which **slopes upwards from left to right** has a **positive** gradient.
- A line which **slopes downwards from left to right** has a **negative** gradient.
- The gradient or slope of horizontal line is **0** while the gradient of vertical line is **undefined**.

Now look at the examples of lines below.



- Which numbered lines have a positive slope? **Answer: 2 and 4**
- Which numbered lines have a negative slope? **Answer: 1 and 3**
- What slope has the horizontal line 5? **Answer: 0**
- What slope has the vertical line 6? **Answer: It does not have a slope**

Let us recall finding the slope or gradient of a straight line.

Example 1

Consider the equation of the straight line $y = \frac{2}{3}x - 4$. To find the slope or gradient, we need two points.

Let us choose two values for x and solve for each corresponding y .

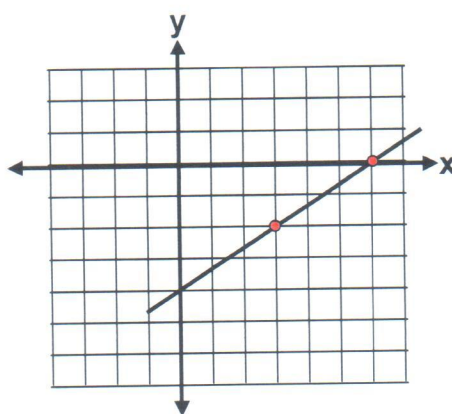
We choose the x -values to be multiples of 3 because the denominator of the fraction is 3. Hence,

$$\text{If } x = 3, \text{ then } y = \left(\frac{2}{3}\right)(3) - 4 = 2 - 4 = -2.$$

$$\text{If } x = 6, \text{ then } y = \left(\frac{2}{3}\right)(6) - 4 = 4 - 4 = 0.$$

So the two points $(3, -2)$ and $(6, 0)$ are on the line $y = \frac{2}{3}x - 4$.

Its graph looks like this:



To find the slope, you use the following formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

If we let point $(3, -2)$ as P_1 and $(6, 0)$ as P_2 , then $x_1 = 3$, $y_1 = -2$, $x_2 = 6$, and $y_2 = 0$.

Substituting these in the formula, we have:

$$m = \frac{-2 - 0}{3 - 6} = \frac{-2}{-3} = \frac{2}{3}$$

Therefore the gradient of the line is $\frac{2}{3}$.

Example 2

Calculate the slope of the line that passes through the points $(1, 2)$ and $(3, 6)$.

Solution: To solve this problem, use the slope formula. $m = \frac{y_1 - y_2}{x_1 - x_2}$

Therefore, the slope of the line passing through points $(1, 2)$ and $(3, 6)$ is:

$$m = \frac{2 - 6}{1 - 3} = \frac{-4}{-2} = 2$$

Note: It does not matter which point you call the first and which the second. But if you calculate the change in y starting with $(1, 2)$, then you must calculate change in x also starting with $(3, 2)$.

The slope (**m**) 2 means that on the straight line joining those two points, for every one unit the value of x changes, the value of y will change by 2 units. We call this the **rate of change** of y with respect to x . It is 2 for every 1.

Example 3

Calculate the slope of the straight line passing through the points $(2, -9)$ and $(-2, -5)$.

Solution: Use the slope formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

Substituting the values: $m = \frac{-9 - (-5)}{2 - (-2)}$

$$m = \frac{-4}{4}$$

$$m = -1$$

Therefore, the slope of the line passing through the points $(2, -9)$ and $(-2, -5)$ is **-1**.

Example 4

Find the slope of a line that passes through the points $(4, 8)$ and $(-6, 3)$.

Solution: Use the slope formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

Substituting the values: $m = \frac{8 - (3)}{4 - (-6)}$

$$m = \frac{5}{10}$$

$$m = \frac{1}{2}$$

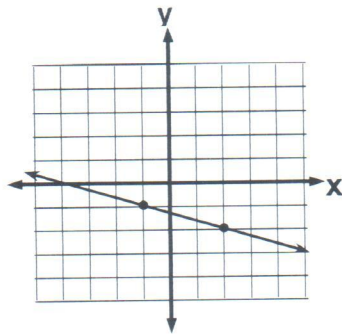
Therefore, the slope of a line that passes through the points $(4, 8)$ and $(-6, 3)$ is **$\frac{1}{2}$** .

NOW DO PRACTICE EXERCISE 10

**PRACTICE EXERCISE 10**

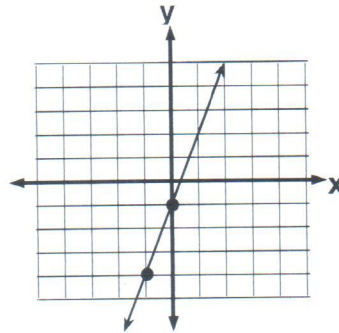
1. Find the slope of each line.

a.



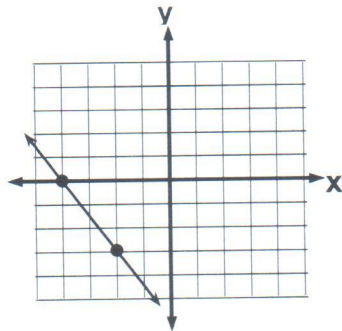
Answer: _____

b.



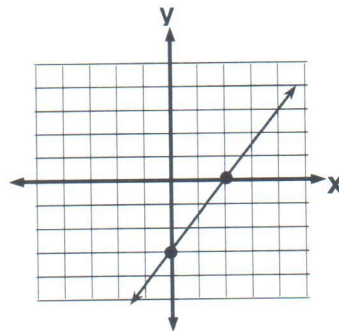
Answer: _____

c.



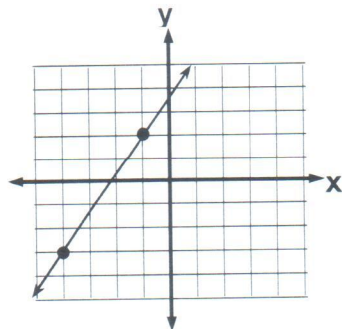
Answer: _____

d.



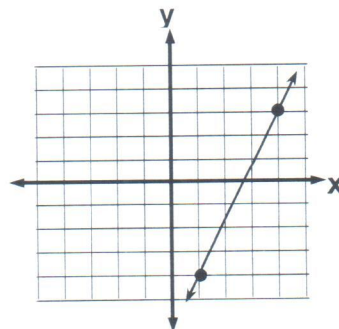
Answer: _____

e.



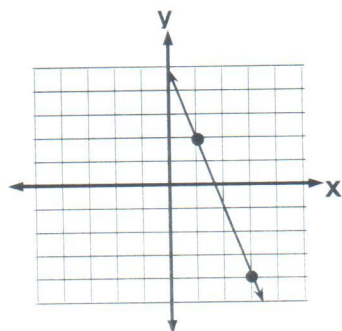
Answer: _____

f.



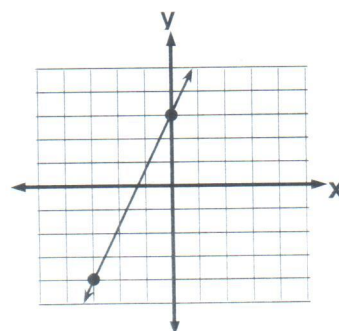
Answer: _____

g.



Answer: _____

h.



Answer: _____

2. Find the slope of the line through each pair of points.

a) $(-1, 4)$ and $(3, 1)$

Answer: _____

b) $(4, 8)$ and $(-6, 3)$

Answer: _____

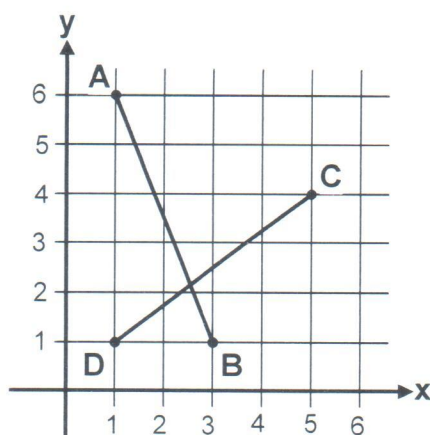
c) $(2, 3)$ and $(5, 5)$

Answer: _____

d) $(4, 2)$ and $(6, 8)$

Answer: _____

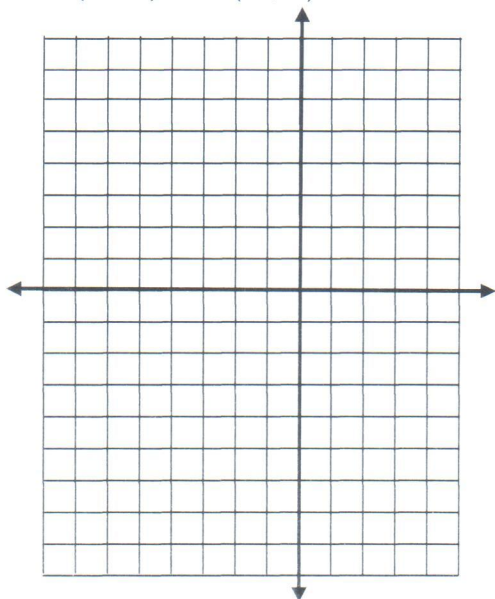
3. Find the slope of the lines AB and CD.



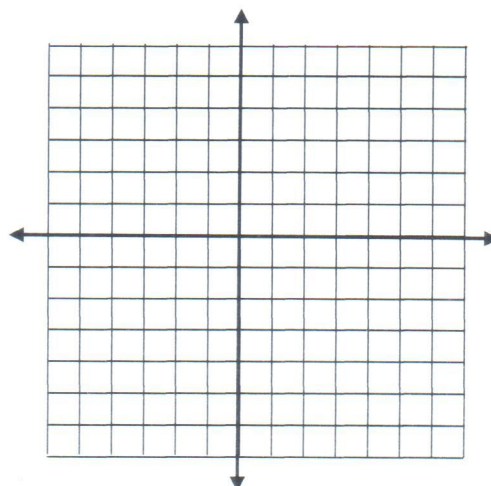
Answers: _____

4. Sketch each pair of coordinate points and find the slope of the straight line joining them.

a) $(-4, 7)$ and $(-7, 4)$



b) $(0, 3)$ and $(4, 4)$



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 11: Finding the Equation of a Straight Line



You have learned to find the slope and the intercepts of a straight line in the previous lessons.



In this lesson, you will:

- identify the different forms of a straight line
- find the equation of the line given its slope and its intercepts
- find the equation of the line given its slope and a point lying on it
- find the equation of the line given two points
- give the equation of the line in either of the forms $y = mx + b$ or $ax + by + c$.

You can find the equation of a straight line, when you are given some information about the line. The information could be the value of its gradient, together with the coordinates of a point on the line. Alternatively, the information might be the coordinates of two different points on the line.

There are many different ways of writing the final equation, and some are more general than others.

The most general form of an equation of a line is

$$Ax + By + C = 0$$

Where **A**, **B** and **C** are real numbers, **A** and **B** are not both **0**.

The equation is usually written in simplified form with **A** positive.

In this lesson, we will consider three additional forms: the **point-slope** form, the **slope-intercept** form, and the **two-point** form.

All these forms depend directly or indirectly on the idea of slope of a line.

The Point-slope form

To find the equation of a line, which has a slope **m** and passes through the point (x_1, y_1) , substitute into the formula

$$y - y_1 = m(x - x_1)$$

Example 1

Find the point-slope form of the equation of a straight line which has a slope of $\frac{1}{4}$ and passes through the point (2, 3). Also write the general form of the equation of this line.

Solution: Given: $m = \frac{1}{4}$ and $P_1 = (x_1, y_1) = (2, 3)$

Substituting these values to the formula;

$$y - y_1 = m(x - x_1)$$

We have: $y - 3 = \frac{1}{4}(x - 2)$ ← **Point-slope form**

To obtain the general form of the equation, eliminate fractions, clear the parentheses, and collect like terms on the left side of the equation.

$$y - 3 = \frac{1}{4}(x - 2)$$

$$4(y - 3) = 4\left[\frac{1}{4}(x - 2)\right] \quad \text{Multiply by LCD} = 4$$

$$4y - 12 = x - 2 \quad \text{Remove parentheses}$$

$$4y - 12 - x + 2 = 0 \quad \text{Transposition}$$

$$-x + 4y - 10 = 0 \quad \text{Collect like terms}$$

$$x - 4y + 10 = 0 \quad \text{Multiply by -1}$$

Therefore the general form is **$x - 4y + 10 = 0$** .

Example 2

Find the general form of a line passing through the point (0, -2) and slope 3.

Solution: We are given a point (0, -2) and slope 3. Substitute these in the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 3(x - 0)$$

$$y + 2 = 3x$$

$$0 = 3x - y - 2$$

$$3x - y - 2 = 0$$

Therefore the general form is **$3x - y - 2 = 0$**

The point-slope form enables us to find the equation of a line when we are given a point on the line and the slope of the line. A special case occurs when the given point is the y-intercept.

The Slope-intercept form(The Gradient –intercept form)

To find the equation of a line, that has a slope **m** and y-intercept (0, b), substitute into the formula

$$y = mx + b$$

To see how this formula is obtained, use the y-intercept (0, b) as the given point in the point-slope form, and substitute to obtain the slope- intercept form.

$$y - y_1 = m(x - x_1)$$

$$y - b = m(x - 0)$$

$$y - b = mx$$

$$y = mx + b$$

Example 3

Find the slope-intercept form of the equation of a line with a slope -2 and y-intercept of (0, 7).

Solution: Substituting -2 for **m** and 7 for **b** in the formula,

$$y = mx + b$$

We obtain, $y = -2x + 7$

Therefore, the slope-intercept form of the equation is **$y = -2x + 7$** .

Example 4

Find the equation of the line that has a slope $\frac{2}{3}$ and a y-intercept of 2. Also write the general form of the equation of this line.

Solution: Substituting $\frac{2}{3}$ for **m** and 2 for **b** in the formula,

$$y = mx + b$$

We obtain, $y = \frac{2}{3}x + 2$ ← **Slope-intercept form**

To obtain the general form of the equation, eliminate fractions

$$3(y) = 3(\frac{2}{3}x + 2)$$

$$3y = 2x + 6$$

$$2x - 3y + 6 = 0$$
 ← **General form**

Example 5

Find the slope and y-intercept of a line with the equation $2x - 3y + 5 = 0$.

Solution: First solve for y to obtain the slope-intercept form

$$2x - 3y + 5 = 0$$

$$-3y = -2x - 5$$

Subtract 2x and 5.

$$-\frac{1}{3}(-3y) = -\frac{1}{3}(-2x - 5)$$

Multiply by $-\frac{1}{3}$.

$$y = \frac{2}{3}x + \frac{5}{3}$$

Remove parentheses.

$$m = \frac{2}{3} = \text{slope}$$

$$(0, \frac{5}{3}) = \text{y-intercept } (\frac{5}{3} = b)$$

Therefore, the slope is $\frac{2}{3}$ and the y-intercept is $(0, \frac{5}{3})$.

The third form that we consider involves finding the equation of the line when we are given two points on that line.

The Two-point form

To find the equation of a line, that passes through the two distinct points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ substitute into the formula

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

To see how this is obtained, the two point form of the equation of a line follows directly from the point-slope form and the definition of the slope of a line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting into $y - y_1 = m(x - x_1)$,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{Substitute } \frac{y_2 - y_1}{x_2 - x_1} \text{ for } m.$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \quad \text{Divide both sides by } y_2 - y_1$$

Example 5

Find the equation of a straight line in general form passing through the points (4, -1) and (2, 7).

Solution: We let $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (2, 7)$ and substitute into the two-point form.

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{7 - (-1)} = \frac{x - 4}{2 - 4}$$

$$\frac{y + 1}{7 + 1} = \frac{x - 4}{-2}$$

$$-2(y + 1) = (x - 4)(8)$$

$$-2y - 2 = 8x - 32$$

$$y + 1 = 4x + 16$$

$$4x + y - 15 = 0$$

The general form of the equation of the line passing through (4, -1) and (2, 7) is **$4x + y - 15 = 0$** .

Example 6

Find the general form of the equation of a line passing through the points (1, -2) and (-3, 0).

Solution: Let $(x_1, y_1) = (1, -2)$ and $(x_2, y_2) = (-3, 0)$ and substitute into the formula then re-arrange.

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-2)}{0 - (-2)} = \frac{x - 1}{-3 - 1}$$

$$\frac{y + 2}{2} = \frac{x - 1}{-4}$$

$$-4(y + 2) = 2(x - 1)$$

$$-4y - 8 = 2x - 2$$

$$-2y - 4 = x - 1$$

$$x + 2y + 3 = 0$$

The general form of the equation of the line passing through (4, -1) and (2, 7) is **$x + 2y + 3 = 0$** .

The names of the forms of the equations identify the quantities necessary for determining each equation.

- The **point-slope form** requires knowing a point and the slope.
- The **slope-intercept form** requires knowing the slope and the y-intercept.
- The **two-point form** requires knowing two points on the line.

It is important to memorize each of the forms of the equation of a line and be able to identify which form applies in a given situation.

NOW DO PRACTICE EXERCISE 11



Practice exercise 11

- Find the general equation of the line that has a slope 3 and passing through the point $(1, 4)$.
- Find the general equation of a line passing through the points $(7, -1)$ and $(5, 3)$.
- A line has the points $(-2, 3)$ and $(-1, -3)$. Find the equation in slope-intercept form. What is the slope? the y-intercept?
- Find the general equation of the line passing through the point $(3, -1)$ with a slope -2 .
- Find the slope and the y-intercept of the line with equation $5x - 2y + 4 = 0$.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 12: More Sketches of Straight Lines



You learnt the different names of the forms of the equation of a line in the previous lesson. You also learnt to find the equation of a line given some information about the line.



In this lesson, you will:

- revise what travel graphs
- read and interpret travel graphs
- solve problems involving graphs

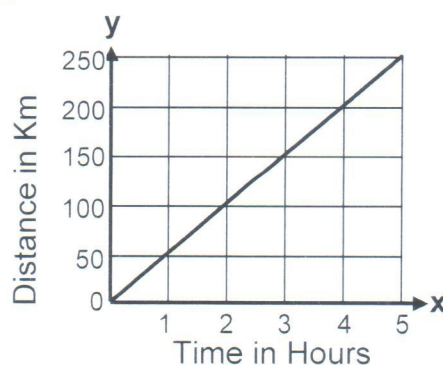
Let us start the lesson by recalling what we learnt about travel graphs

A graph that shows the distance travelled by an object and over a period of time is called a **travel graph**.

The time taken to travel a certain distance is always shown on the horizontal axis of the graph and the distance travelled is always shown on the vertical axis of the graph.

The time taken is measured in hours and the distance travelled is normally measured in kilometers.

Below is an example of travel graph.



A straight line graph on a travel graph indicates a constant speed.

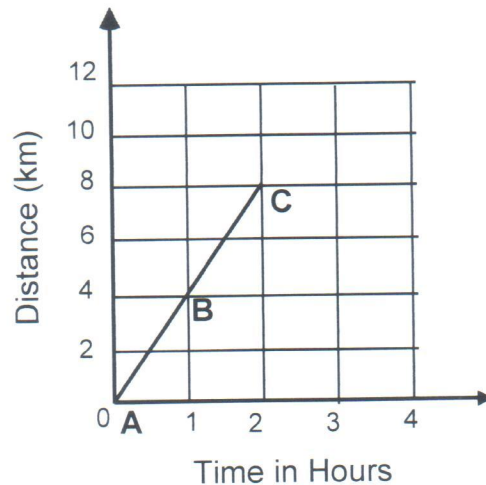
The **speed** is the distance covered in a certain amount of time.

The speed is calculated using the formula:

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \text{ or } S = \frac{D}{T}$$

For example, the speed of a car might be 50 km per hour, the speed of a boat might be 20 m per minute, a bird flying at 920 m per min or a plane flying might be 500 km/h.

Shown below is a travel graph. It is for a walker who walks steadily at a speed of 4 km per hour for 2 hours. Starting at a place **A**, he reaches place **B** after one hour and ending at place **C** after 2 hours.



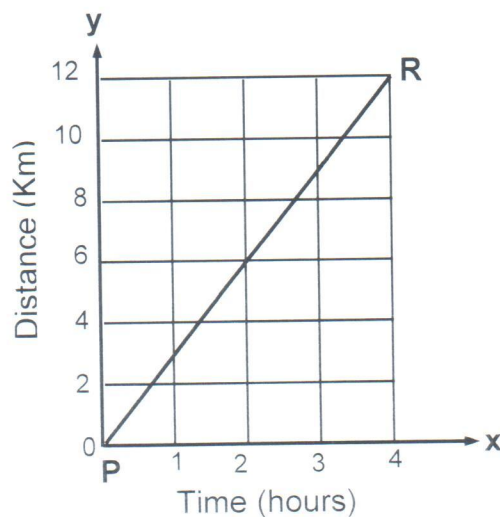
The graph looks like this because;

- (i) when starting off, the time is **zero**, and the distance travelled is **zero**, therefore the co-ordinates at this point **A** is (0,0)
- (ii) after 1 hour, he has travelled 4 km, so the co-ordinates at this point **B** is (1,4)
- (iii) after 2 hours he has travelled 8 km, so the co-ordinates at this point **C** is (2,8).

We can calculate the speed from a given travel graph.

Example 1

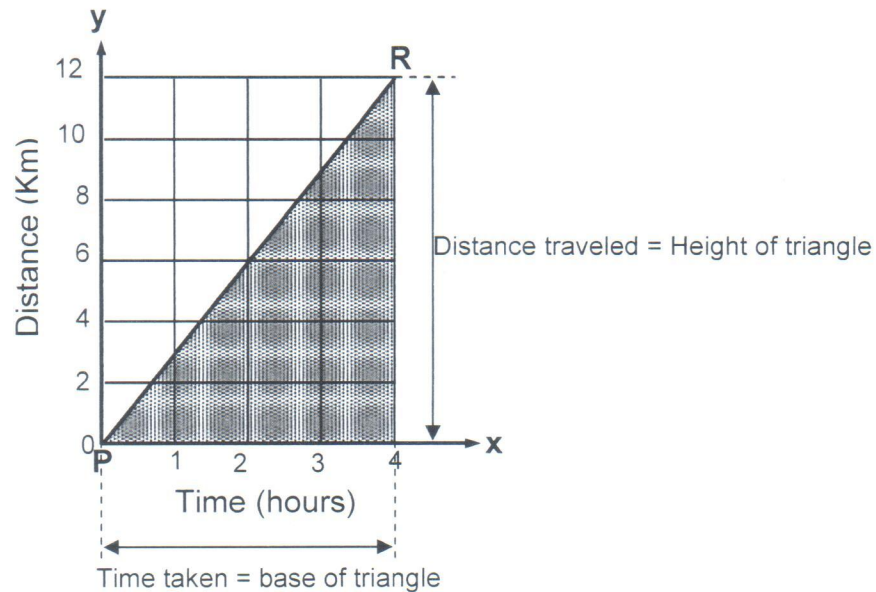
Study the travel graph below. It shows the journey of a walker starting at **P** and finishing at **R**.



How can we work out the speed of the walker from the graph?

Solution:

First, let us shade the triangle below the line **PR** as shown below:



Notice that the **height** of the shaded triangle equals the **distance travelled** and the **base** of the triangle equals the **time taken**.

Hence,
$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\text{height of triangle}}{\text{length of base}}$$

But you know from what you have already learnt in Lesson 10 that the ratio:

$$\frac{\text{height of triangle}}{\text{length of base}} = \frac{\text{Change in } y}{\text{Change in } x} \text{ which is the slope or gradient of the travel line.}$$

Therefore, for a travel graph:

Speed of an object = Slope or Gradient of the travel line of the object.

So referring to the shaded triangle, we have:

height of triangle = 12 km and length of base = 4 hours

$$\text{Therefore, speed} = \frac{12 \text{ km}}{4 \text{ h}} = 3 \text{ km/h}$$

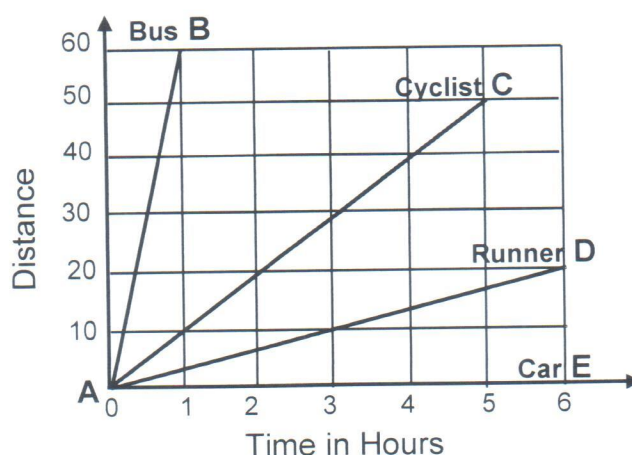
The speed from **P** to **R** on any journey can be found from a travel graph by finding the gradient of the travel line by using these steps:

1. find the distance travelled from **P** to **R** by referring to the increase in distance on the vertical axis.
2. find the time taken from **P** to **R** by referring to the increase in time on the horizontal axis.
3. divide the distance found in step 1 by the time found in step 2.

Example 2

The travel graph below shows the journeys of a bus (line **AB**), a cyclist (line **AC**), a runner (line **AD**), and a car (line **AE**).

TRAVEL GRAPH



Find the speed of the:

- a. bus b. cyclist c. runner d. car

Solution:

Work out the different speeds by first finding the distance travelled by referring to the increase in distance on the vertical axis and then finding the time taken by referring to the increase in time on the horizontal axis. The speeds are then found by dividing the distance travelled by the time taken at convenient points on the graphs in each case.

a. Bus: Speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{60 \text{ km}}{1 \text{ h}} \quad \leftarrow \text{Since the distance travelled from A to B} = 60 \text{ km and the time taken from A to B} = 1 \text{ hour.}$$

$$= 60 \text{ km/h}$$

b. Cyclist: Speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{50 \text{ km}}{5 \text{ h}} \quad \leftarrow \text{Since the distance travelled from A to C} = 50 \text{ km and the time taken from A to C} = 5 \text{ hours.}$$

$$= 10 \text{ km/h}$$

c. Runner: Speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{20 \text{ km}}{6 \text{ h}} \quad \leftarrow \text{Since the distance travelled from A to D} = 20 \text{ km and the time taken from A to D} = 6 \text{ hours.}$$

$$= 3\frac{1}{3} \text{ km/h}$$

d. Car: $\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{0 \text{ km}}{6 \text{ h}} \quad \leftarrow \text{Since distance travelled from A to E} = 0 \text{ km (the line AE is horizontal) and time taken from A to E}$$

$$= 0 \text{ km/h} \quad \leftarrow \text{Speed is zero - the car must be parked since it hasn't travelled any distance at all in the 6 hours!}$$

Notice the travel graph for the bus, which has the biggest speed of 60 km/h. This is the steepest and has the biggest slope of the four graphs.

The travel graph for the car, which has the least speed of 0 km/h, which means, it has not moved any distance. This is the least steep and has the smallest slope.

Generally, for any travel graph containing more than one object travelling:

1. The smaller the slope of a travel graph, the smaller the speed, the bigger the slope of a line, the bigger the speed.
2. If the line is horizontal, the speed is zero.

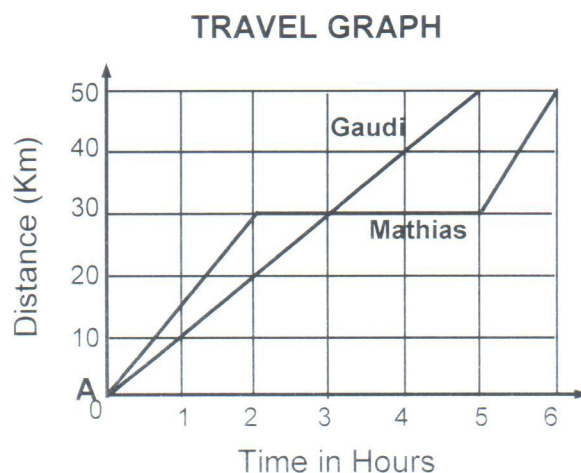
We can also find the average speed of an object from a travel graph.

Average speed is the ratio of the total distance traveled by an object to the total time the object taken during the journey.

$$\text{Average Speed} = \frac{\text{Total distance traveled}}{\text{Total time taken}}$$

Example 3

Look at the travel graph below of two cyclists, Mathias and Gaudi in a 50 kilometre race. They both started off at the same time at place **A**. Unfortunately, during the race, Mathias has a flat tyre which takes a long time to fix. As soon as he repaired the tyre he continued the race.



- What is Gaudi's average speed in the race?
- What is Mathias' average speed in the race?

Solutions:

a. Gaudi's average Speed

$$\text{Gaudi's average speed for the whole race} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Given: total distance of the race = 50 km

Gaudi's total time of the race = 5 hours

$$\text{So, Gaudi's average speed for the whole race} = \frac{50 \text{ km}}{5 \text{ h}}$$

$$= 10 \text{ km/h}$$

(b) Mathias' average speed

$$\text{Mathias' average speed for the whole race} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{50}{6}$$

$$= 8\frac{1}{3} \text{ km/h.}$$

We can answer further questions using the above graph.

c. How long did it take for Mathias to repair the tyre?

The horizontal part of Mathias' travel line shows us when he stopped to repair the tyre. He had the flat tyre after cycling 2 hours and the tyre was repaired in 5 hours after the start of the race.

$$\begin{aligned}\text{Then time to repair tire} &= 5 \text{ hours} - 2 \text{ hours} \\ &= 3 \text{ hours.}\end{aligned}$$

d. When and where did Gaudi overtake Mathias?

Gaudi overtook Mathias at the point where the travel lines cross.

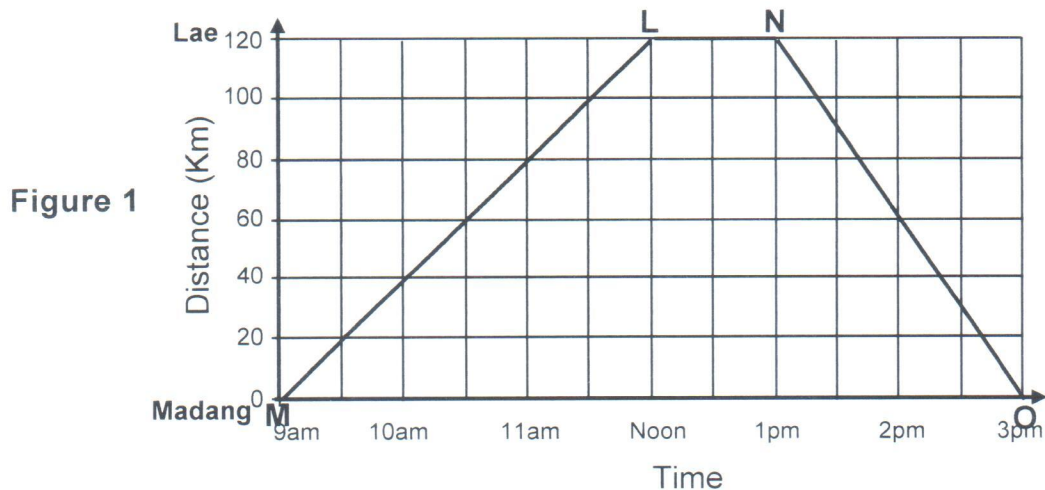
Hence, Gaudi overtook Mathias **3 hours** after the start of the race and they were **30 km** from **A** when Gaudi overtook Mathias.

The next example we will look at the negative slope or gradient of travel graphs.

Example 4

Shown below is a travel graph (fig.1) for a car which starts at Madang (**M**) at 9 a.m., arrives at Lae at 12 noon (**L**), stays at Lae for one hour and then at 1 p.m. (**N**) returns to Madang, arriving at 3 p.m. (**O**).

TRAVEL GRAPH OF A JOURNEY FROM MADANG TO LAE AND BACK



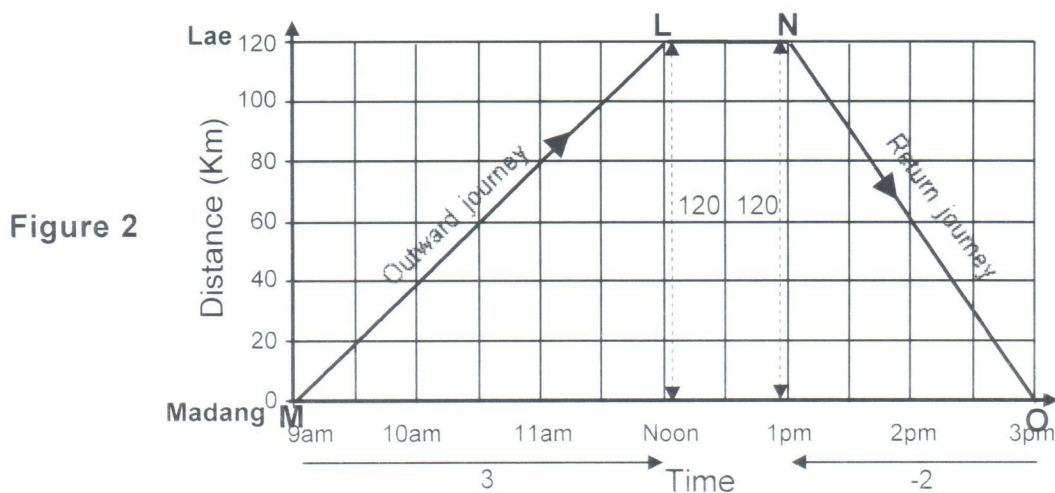
- Work out the speed on the **outward** journey from Madang **M** to Lae **L**
- Workout the speed on the return journey from Lae **L** to Madang **M**.

Solution:

First let us work out the speed on the outward journey from Madang to Lae.

Notice that the travel line **ML** for the outward journey leans to the **right**, which is the speed. The slope of the line will be **positive** (see Figure 2 below).

TRAVEL GRAPH OF A JOURNEY FROM MADANG TO LAE AND BACK



From the graph: Distance travelled = 120 km,
time taken = 3 hours

So, to find the speed on the **outward** journey from Madang to Lae we have:

$$\begin{aligned}\text{Speed from M to L} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{120}{3} \text{ km/h} \\ &= 40 \text{ km/h}\end{aligned}$$

- b. The speed on the **return** journey from Lae **N** to Madang **O**.

Notice that the travel line **NO** for the return journey leans to the **left** and therefore the speed or the slope of the line will be **negative** as referring to Figure 2.

From the graph: Distance travelled = 120 km, time taken = -2 hours.

So, to find the speed on the **return** journey from Lae to Madang we have:

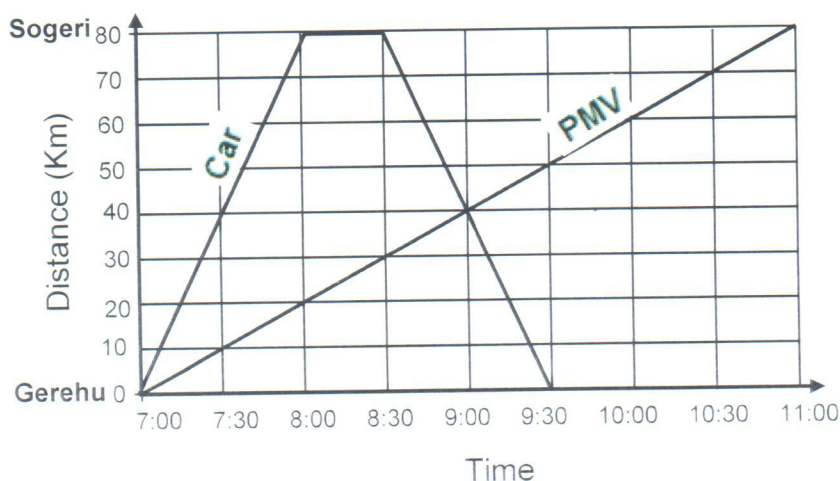
$$\begin{aligned}\text{Speed from N to O} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{120}{-2} \text{ km/h} \\ &= -60 \text{ km/h}\end{aligned}$$

Because the speed is negative, this tells us that the car on its return journey is travelling in the **opposite** direction to that on its outward journey, which is the positive direction.

Example 5

The travel graph below shows the journey of a car, which travels from Gerehu to Sogeri, stopping in Sogeri for $\frac{1}{2}$ hour and then returning to Gerehu; also, the graph shows the journey of a PMV, travelling from Gerehu to Sogeri on the same road as the car. Both the car and the PMV leave Gerehu for Sogeri at 7 a.m.

TRAVEL GRAPH OF A CAR AND A PMV TRAVELLING BETWEEN GEREHU AND SOGERI



Let us answer the following questions.

- a. What is the speed of the PMV as it travels from Gerehu to Sogeri?
- b. What is the speed of the car as it travels from Gerehu to Sogeri?
- c. What is the speed of the car on its outward journey from Gerehu to Sogeri?
- d. What is the speed of the car on its return journey from Sogeri to Gerehu?
- e.
 - i. At what time do the car and the PMV meet each other on the road?
 - ii. How far are the car and the PMV from Gerehu when they meet?

Solutions:

- a. Speed of the PMV as it travels from Gerehu to Sogeri?

$$\begin{aligned}\text{Speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{80}{4} \text{ km/h} = 20 \text{ km/h}\end{aligned}\quad \textbf{Answer: 20 km/h}$$

- b. Speed of the car as it travels from Gerehu to Sogeri?

$$\begin{aligned}\text{Speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{80}{1} \text{ km/h} = 80 \text{ km/h}\end{aligned}\quad \textbf{Answer: 80 km/h}$$

- c. Speed of the car on its outward journey from Gerehu to Sogeri?

$$\begin{aligned}\text{Speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{80}{1} \text{ km/h} = 80 \text{ km/h}\end{aligned}\quad \textbf{Answer: 80 km/h}$$

- d. What is the speed of the car on its return journey from Sogeri to Gerehu?

$$\begin{aligned}\text{Speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{80}{-1} \text{ km/h} = -80 \text{ km/h}\end{aligned}\quad \textbf{Answer: -80 km/h}$$

- e.
 - i. Time when the car and the PMV meet each other on the road?

They meet at 9 a.m. **Answer: 9 a.m.**

- ii. Distance of the car and the PMV from Gerehu when they meet?

Distance = 40 km

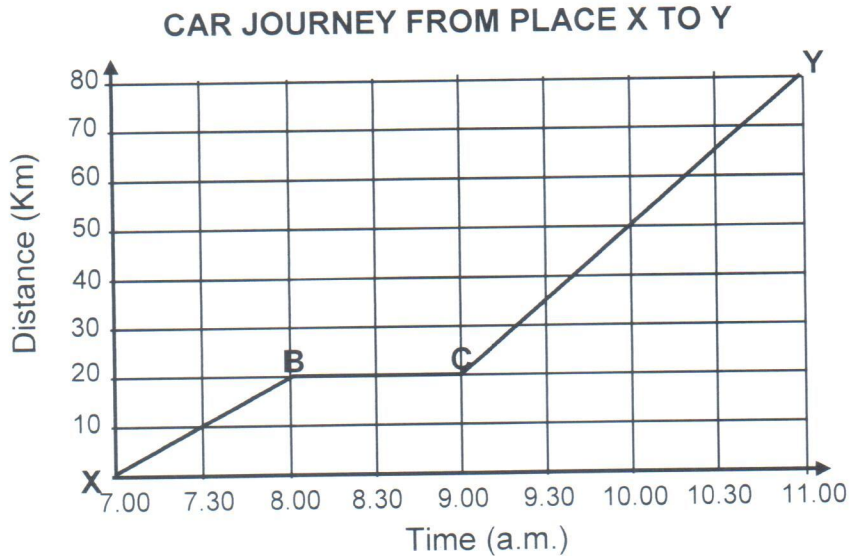
Answer: 40 km

NOW DO PRACTICE EXERCISE 12



Practice exercise 12

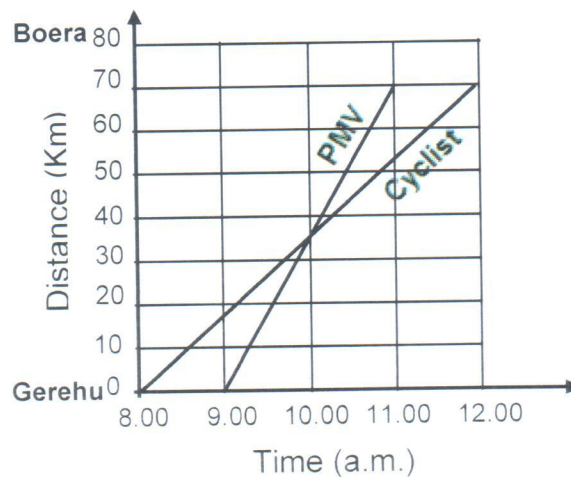
1. Below is a travel graph of a car journey. The car starts from place **X** at 7.00 a.m. and finally arrives at place **Y** at 11.00 a.m., having travelled 80 kilometres altogether. What is the average speed for the whole journey?



Answer: _____

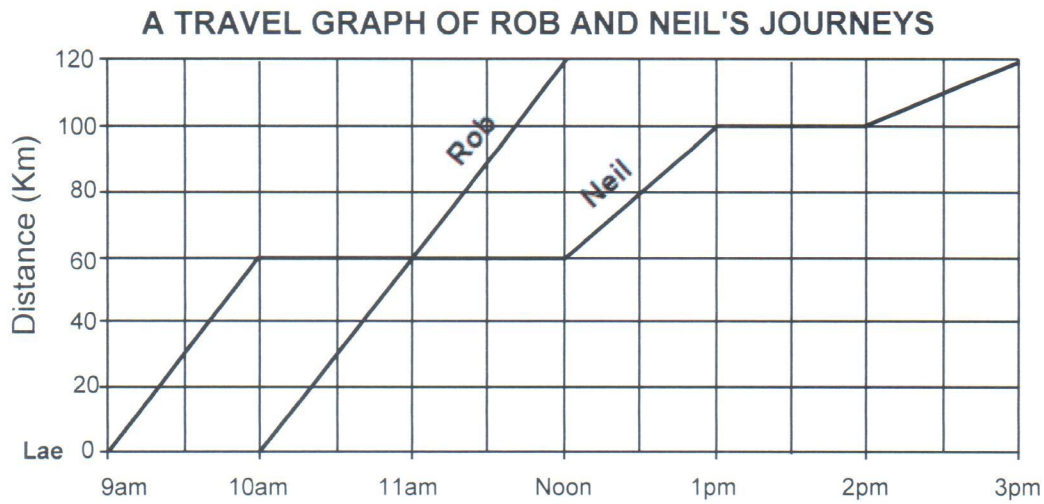
2. Look at this travel graph, which shows the journeys of a cyclist and a PMV both travelling on the main road from Gerehu to Boera. At what time and after covering what distance do they meet?

A CYCLIST'S AND A PMV'S JOURNEY FROM GEREHU TO BOERA



Answer: _____

3. The graph below shows the journeys from Lae of two motorists Rob and Neil. They both travel on the same road.



Study the graph carefully and answer the questions below.

- (a) At what time do the two motorists meet?

Answer: _____

- (b) How far from Lae are they when they meet?

Answer: _____

- (c) What is Bob's average speed for the journey?

Answer: _____

- (d) (i) How much time altogether (including the time when he was stationary) did Neil take for the whole journey?

Answer: _____

- (ii) What was Neil's average speed for the whole journey?

Answer: _____

- (e) (i) Between what times was Neil travelling fastest?

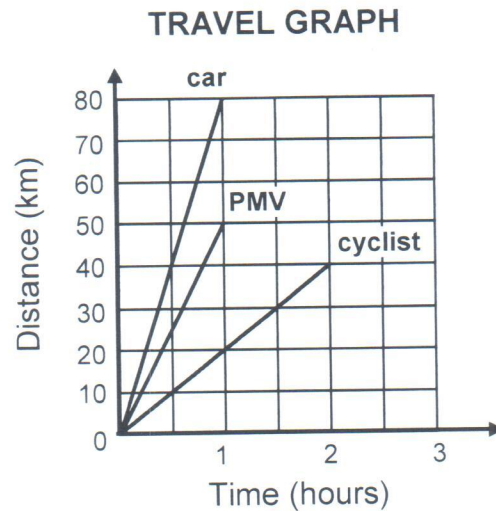
Answer: _____

- (ii) What was his speed during this time?

Answer: _____

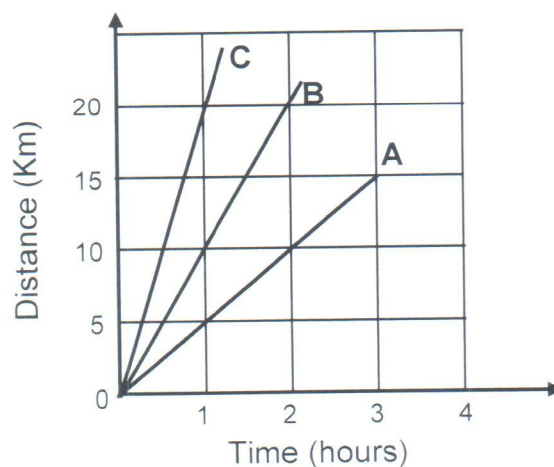
4. On the grid drawn here and label line graphs to show the distance travelled in one hour by a:

- (a) PMV travelling at 50 km/h for 1 hour.
 a. Car travelling at 80 km/h for 1 hour.
 (c) Cyclist travelling at 20 km/h for 2 hours.



5. Below are travel graphs for a man **A** walking, a boy **B** running and a cyclist **C** cycling. They all start from the same place and at the same time and go in the same direction. Find the speed of each person.

TRAVEL GRAPH OF A MAN, A BOY AND A CYCLIST



Answers: A: _____ B: _____ C: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

TOPIC 2: SUMMARY



This summarizes the important concepts and ideas to be remembered.

- **Rate** means any single item or unit.
- Rate graph is a graph showing the amount of quantity per unit.
- Conversion graph is a graph showing the relationship of one unit to another.
- The graph of a linear equation is a **straight line**.
- A **table of values** is a graphic organizer or chart that helps to determine two or more points that can be used to create a graph.
- When graphing a line, if the coefficient x in the linear equation is a fraction, choose zero or a multiple of the denominator as your x -coordinates.
- Two points determine a straight line. To graph a straight line is simpler
- **Intercepts** are the points of the line where the line crosses the x and y axes.
 - ❖ **x -intercept** is the point where the line crosses the x -axis.
 - ❖ **y -intercept** is the point where the line crosses the y -axis.
- To find the x -intercept, substitute $y = 0$ in the equation.
- To find the y -intercept, substitute $x = 0$ in the equation.
- When the equation is in the form **$y = ax$** , where $b = 0$, the y intercept is 0. The line crosses through the origin. The x - and y -intercepts coincide.
- When the equation is the form **$x = A$** , where A is constant, the graph is a straight line parallel to the y -axis. The line is **horizontal**.
- When the equation is in the form **$y = A$** , where A is constant, the graph is a straight line parallel to the x -axis. The line is **vertical**.
- The slope of a line is **positive** if the line leans to the right or goes upward from left to right.
- The slope of a line is **negative** if the line leans to the left or goes downward from right to left.
- The slope of a line is **0** if the line is horizontal.
- The slope of a line is **undefined** if the line is vertical.
- Travel graph is a straight line graph which is used to find the time of journeys, distance travelled and speed of an object.
- To find the speed we used the formula:

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- To find the average speed we used the formula:

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

REVISE LESSONS 7 TO 12. THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 4.

ANSWERS TO PRACTICE EXERCISES 7-12

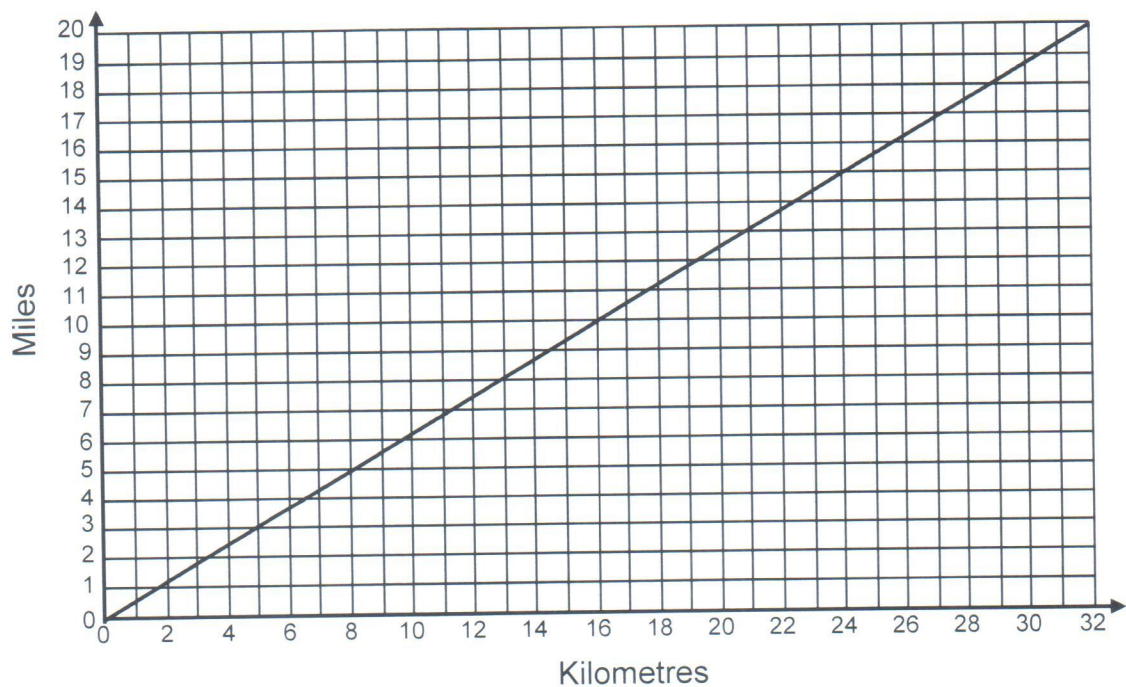
Practice Exercise 7

1. a. K4.20 b. 3000 people/yr.

2. K2.00/hour

3. a. 60% b. 54

4. a. **CONVERSION GRAPH: MILES TO KILOMETRES AND VICE VERSA**

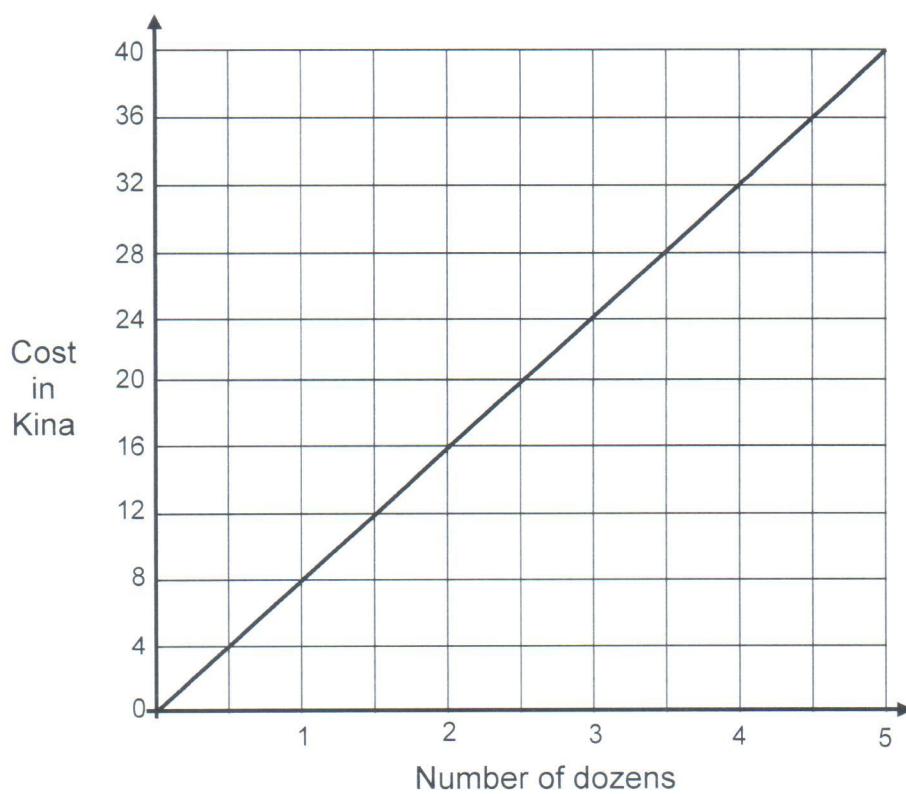


- b. i. 12.5 miles
 ii. 16 km
 iii. 22 km

5. a.

Dozens of eggs	0	1	2	3	4	5
Kina	0	8	16	24	32	40

b.

READY RECKONER: COST OF EGGS PER DOZEN

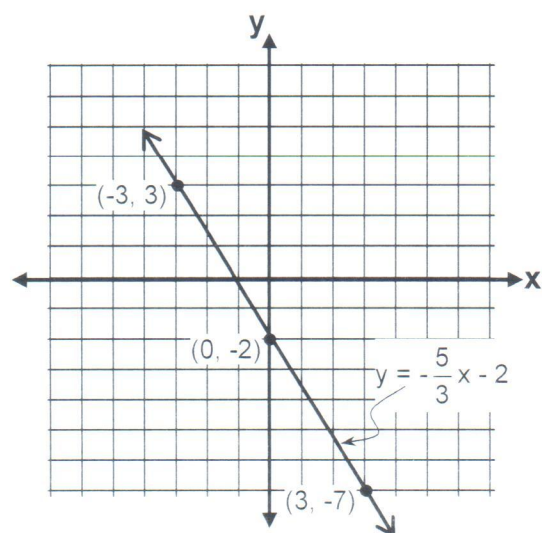
c. i. K24

ii. K36

Practice Exercise 8

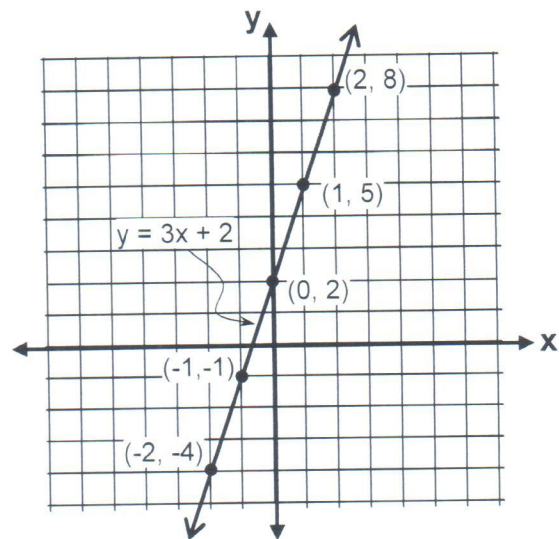
1. a. $y = -\frac{5}{3}x - 2$

x	$-\frac{5}{3}x - 2$	y	Ordered pairs
-3	$-\frac{5}{3}(-3) - 2$	3	(-3, 3)
0	$-\frac{5}{3}(0) - 2$	-2	(0, -2)
3	$-\frac{5}{3}(3) - 2$	-7	(3, -7)



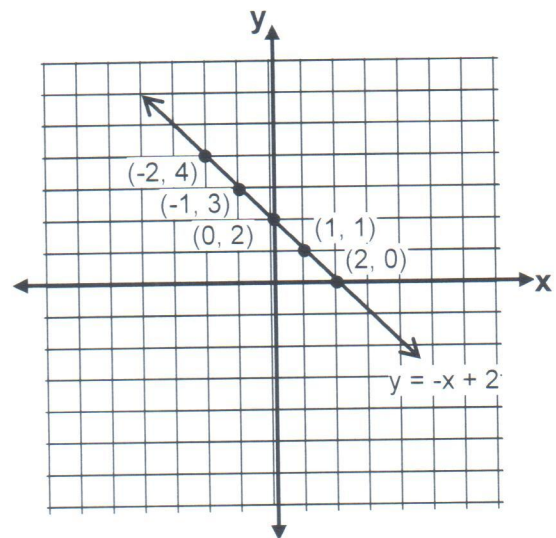
b. $y = 3x + 2$

x	$3x + 2$	y	Ordered pairs
-2	$3(-2) + 2$	-4	$(-2, -4)$
-1	$3(-1) + 2$	-1	$(-1, -1)$
0	$3(0) + 2$	2	$(0, 2)$
1	$3(1) + 2$	5	$(1, 5)$
2	$3(2) + 2$	8	$(2, 8)$



c. $y = -x + 2$

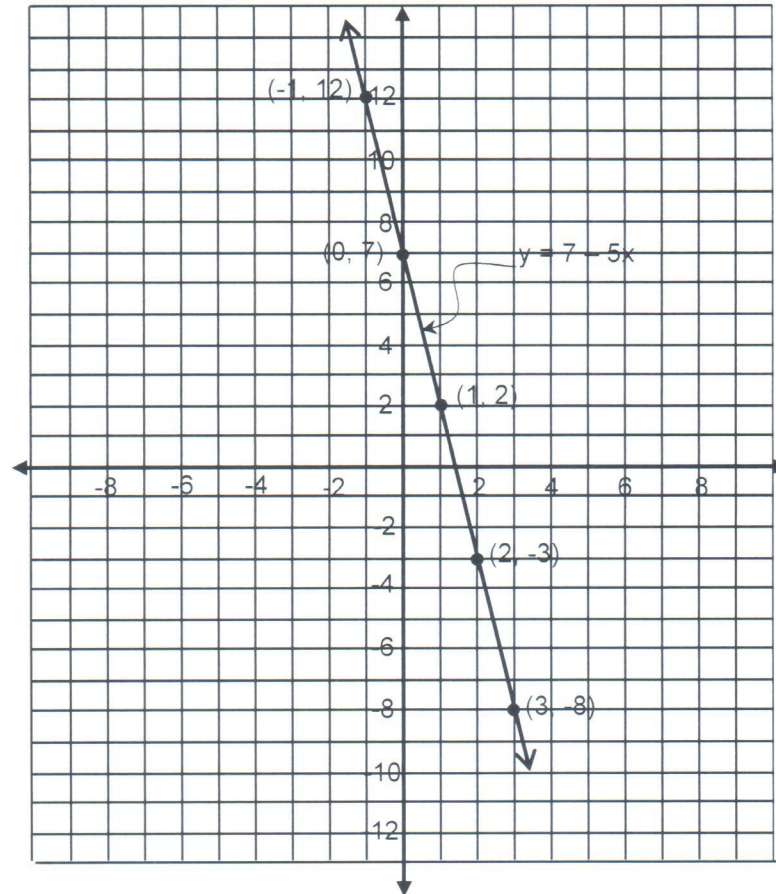
x	$-x + 2$	y	Ordered pairs
-2	$-(-2) + 2$	4	$(-2, 4)$
-1	$-(-1) + 2$	3	$(-1, 3)$
0	$0 + 2$	2	$(0, 2)$
1	$-(1) + 2$	1	$(1, 1)$
2	$-(2) + 2$	0	$(2, 0)$



2. a.

x	$7 - 5x$	y	Ordered pairs
-1	$7 - 5(-1)$	12	$(-1, 12)$
0	$7 - 5(0)$	7	$(0, 7)$
1	$7 - 5(1)$	2	$(1, 2)$
2	$7 - 5(2)$	-3	$(2, -3)$
3	$7 - 5(3)$	-8	$(3, -8)$

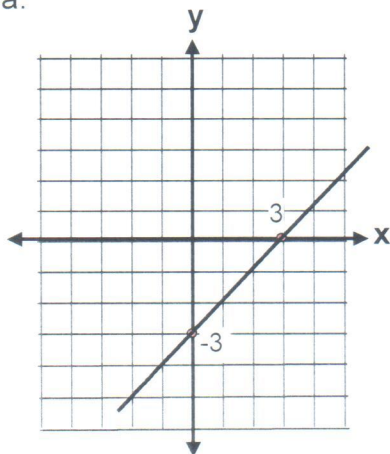
b. The graph



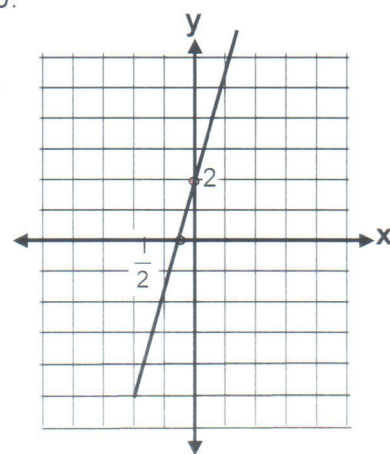
Practice Exercise 9

1. a. $x = -2; y = 4$ b. $x = 4; y = -12$ c. $x = -\frac{1}{4}, y = 1$

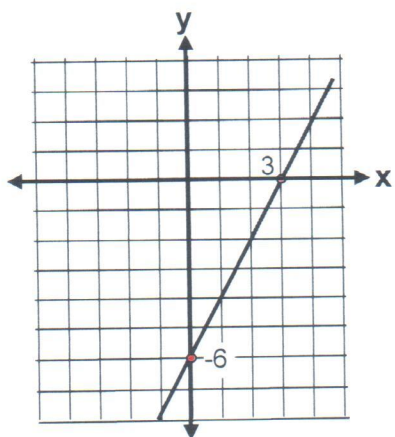
2. a.



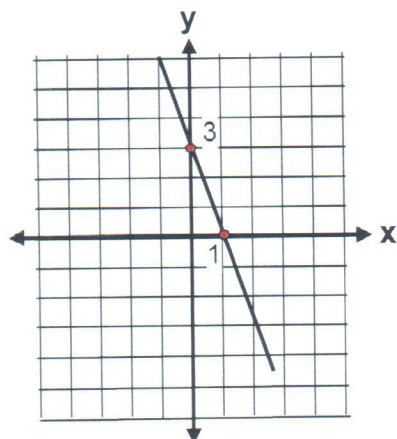
b.



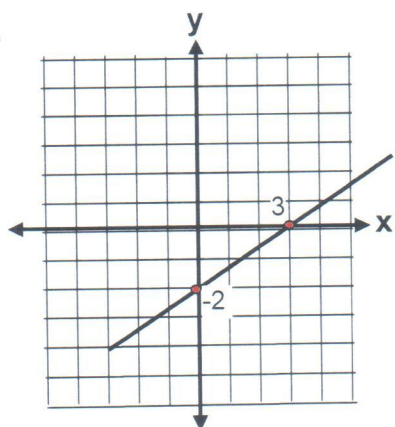
c.



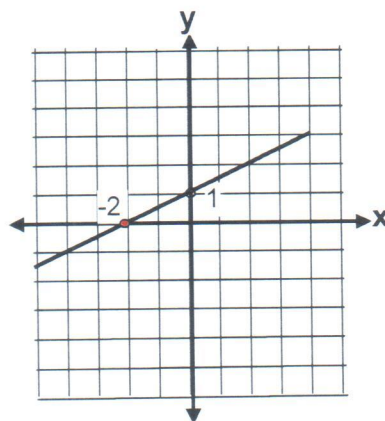
d.



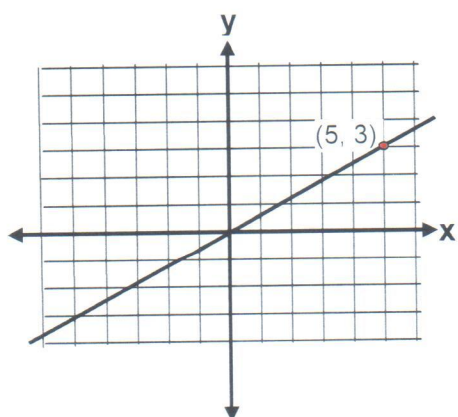
e.



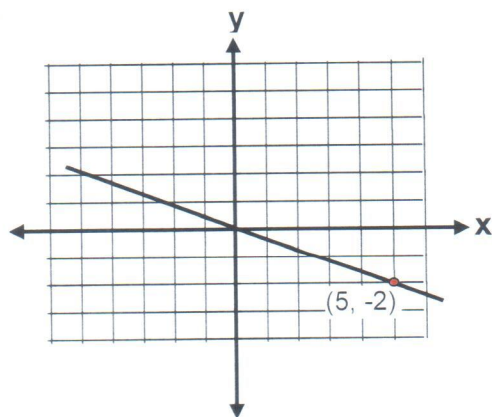
f.



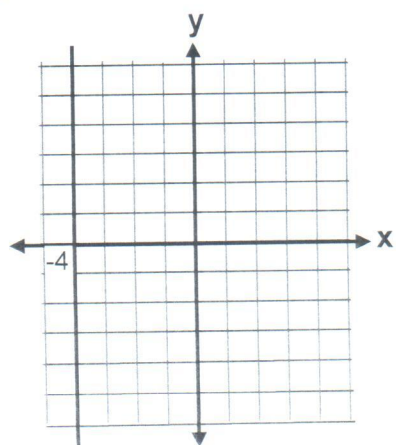
g.



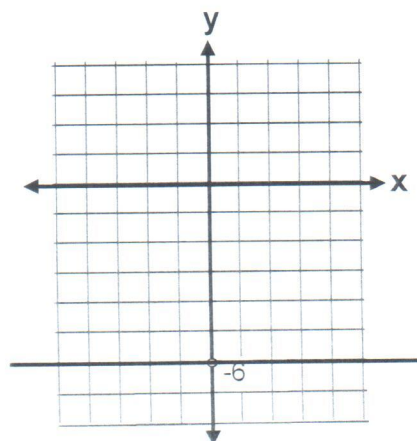
h.



i.



j.



Practice Exercise 10

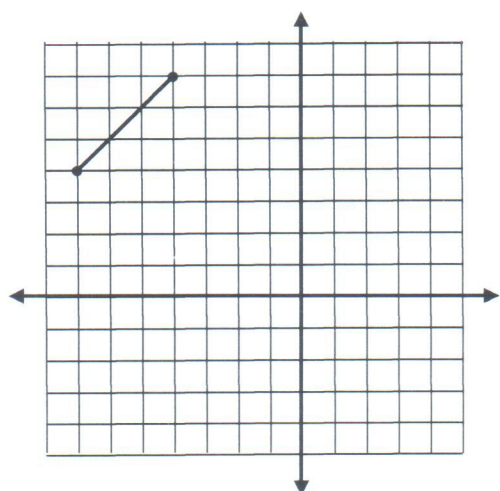
1. a. $-\frac{1}{3}$ b. 3 c. $\frac{3}{2}$ d. $\frac{3}{2}$

e. $\frac{5}{3}$ f. $\frac{7}{3}$ g. -3 h. $\frac{7}{3}$

2. a. $-\frac{3}{4}$ b. $\frac{1}{2}$ c. $\frac{2}{3}$ d. 3

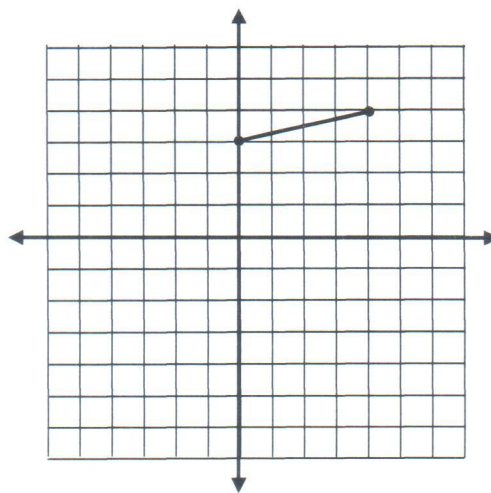
3. Slope of AB = $-\frac{5}{2}$
Slope of CD = $\frac{3}{4}$

4. a. (-4, 7) and (-7, 4)



$m = 1$

b. (0, 3) and (4, 4)



$m = \frac{1}{4}$

Practice Exercise 11

1. $3x - y + 1 = 0$

2. $2x + y - 13 = 0$

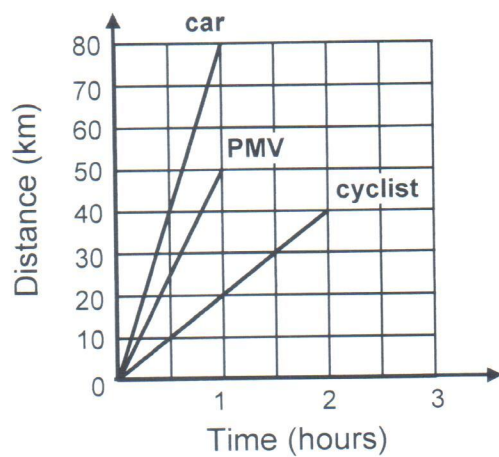
3. slope-intercept form: $y = -6x - 9$; $m = -6$; y-intercept (b) = -9

4. $2x + y - 5 = 0$

5. $m = \frac{5}{2}$; y-intercept(b) = 2

Practice Exercise 12

1. 20 km per hour 2. 10 a.m.; 35 km
3. a. 11a.m. b. 60 km c. 60 km/h d. i. 6 hrs ii. 20 km/h
- 4.



5. a. Man A: 5 km/h
b. Boy B: 10 km/h
c. Cyclist C: 20 km/h

END OF TOPIC 2

TOPIC 3

QUADRATIC EQUATIONS AND PARABOLA

- | | |
|------------|--|
| Lesson 13: | Quadratic Equations |
| Lesson 14: | Solving Quadratic Equations by Factorizing |
| Lesson 15: | Solving Quadratic Equations by Square Root Method |
| Lesson 16: | Solving Quadratic Equations by Completing the Square |
| Lesson 17: | Solving Quadratic Equations by Using the Quadratic Formula |
| Lesson 18: | Graphs of Quadratic Equations |

TOPIC 3: Quadratic Equations and Parabola

Introduction



You have learned about linear equations and their graphs in Topic 2.

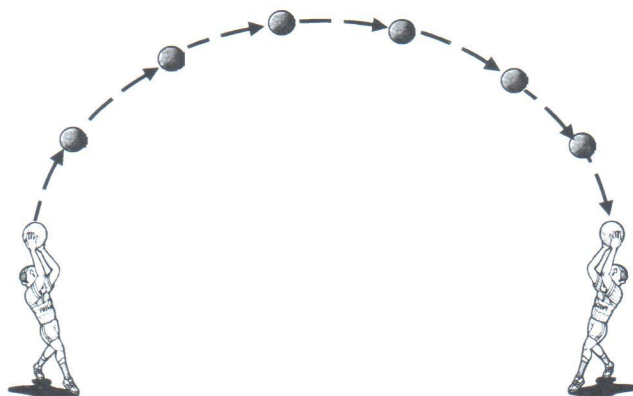
It is not too difficult to realize that not all relationships between two variables will be linear. There are many types of non-linear relationships between two variables and, indeed there may be no observable relationship at all between two variables.

One type of commonly occurring non-linear relationship is represented by the quadratic equation and its graph.

You have seen in Unit 3 that equations containing squared terms as the highest power are called quadratic equations. For example $y = x^2$, $2x^2 + x$, $4x^2 + 3 = 0$ are quadratic equations.

You will discover that when you draw the graph displaying quadratic relationships, we get a curve called a **parabola**.

If you throw a ball (or shoot an arrow, fire a missile or throw a stone) it will go up into the air, slowing down as it goes then come down...the path of the ball through the air, is a parabola as shown.



Parabolas have many practical uses and applications in real world. For example, the curved arches of many buildings, bridges and tunnels have shapes which are parabolas.

In this topic, you will learn to solve quadratic equations, look at their properties and their graphs.

Lesson 13: Quadratic Equations



You have learned the different forms of linear equations and their graphs in Topic 2.



In this lesson, you will:

- identify quadratic equations and their types
- learn and apply the steps in transforming quadratic equations into their standard form.

Let us start this lesson by looking at the following polynomial expressions below.

1. $2x^2 + 16x + 39 = 0$
2. $x^2 - 3x = 0$
3. $5x^2 - 3 = 0$
4. $x^2 = 3x - 1$
5. $x^2 = 16$

Notice that all the above polynomials expressions have second degree term as their highest-degree terms. Such expressions are also called “**second degree equations**” or commonly known as **quadratic equations**.

A **quadratic equation** is a polynomial with a second degree term as its highest-degree term. The name “**quadratic**” comes from “**quad**” meaning “**square**”, because the variable gets squared (like x^2). And because **2** is the largest exponent of **x**, it is also called an “**equation of degree 2**”.

The General Form of Quadratic Equation

The general form a quadratic equation looks like this:

$$ax^2 + bx + c = 0$$

Where: **a, b** and **c** are known values except that **a** $\neq 0$.
x is the variable or the unknown

There are two types of quadratic equation: the complete quadratic and the incomplete or pure quadratic.

A quadratic equation is said to be a **complete quadratic** if it contains both the second power (**a**), the first power of the variable or unknown quantity (**b**) and the constant term (**c**).

- Examples:
1. $2x^2 + 5x + 3 = 0$,
 2. $5x^2 + 3x = -3$
 3. $7x^2 - 2 = 5x$

In example 1, $a = 2$, $b = 5$ and $c = 3$. In example 2, $a = 5$, $b = 3$ and $c = 3$. In example 3, $a = 7$, $b = -5$ and $c = 2$.

- An **incomplete or pure quadratic** is one in which **b** or **c** (or both) is zero. The only letter that cannot be zero is **a**. If **a** is zero, the equation would not be a quadratic.

- Examples:
1. $x^2 - 5x = 0$ -----> ($c = 0$)
 2. $11x^2 - 6 = 0$ -----> ($b = 0$)
 3. $5x^2 = 0$ -----> (b and $c = 0$)

Incomplete quadratic equation is in the form

$ax^2 + bx = 0$

or

$ax^2 + c = 0$

Changing quadratic equation into general form

To change quadratic equations into general form follow the following steps:

1. Remove fractions by multiplying each term by the LCD.
2. Remove grouping symbols.
3. Combine like terms.
4. Get all terms to one side by adding the same expressions to both sides. Only zero must remain on the other side. Then arrange the terms in descending powers.

Example 1

Change the quadratic equation $x^2 = 3x - 1$ into general form.

Solution:

$$x^2 = 3x - 1$$

$$x^2 - 3x = 5 \quad \text{Subtract } 3x \text{ from both sides.}$$

$$x^2 - 3x + 1 = 0 \quad \text{Add 1 to both sides.}$$

Therefore, the general form is $x^2 - 3x + 1 = 0$. (Where: $a = 1$, $b = -3$ and $c = 1$)

Example 2

Change the quadratic equation $3x^2 = 4$ into general form.

Solution:

$$3x^2 = 4$$

$$3x^2 - 0x - 4 = 0 \quad \text{Subtract 4 from both sides.}$$

Therefore, the general form is $3x^2 - 0x - 4 = 0$. (Where: $a = 3$, $b = 0$ and $c = -4$)

Example 3

Change the quadratic equation $9x = 14x^2$ into general form.

Solution:

$$\begin{array}{ll} 9x = 14x^2 & \\ 0 = 14x^2 - 9x & \text{Subtract } 9x \text{ from both sides.} \\ 14x^2 - 9x + 0 = 0 & \text{Add } 0 \text{ to both sides} \end{array}$$

Therefore, the general form is $14x^2 - 9x + 0 = 0$. (Where: $a = 14$, $b = -9$ and $c = 0$)

Sometimes a quadratic equation does not look like any of the forms stated on pages 149 and 150. See examples 4, 5 and 6.

Example 4

Change the quadratic equation $2(2w^2 - 2w) = 5$ into general form.

$$\begin{array}{ll} \text{Solution:} & 2(w^2 - 2w) = 5 \\ & 4w^2 - 4w = 5 \quad \text{Expand (remove brackets)} \\ & 4w^2 - 4w - 5 = 0 \quad \text{subtract } 5 \text{ from both sides} \end{array}$$

Therefore, the general form is $4w^2 - 4w - 5 = 0$. (Where: $a = 4$, $b = -4$, $c = -5$)

Example 5

Change the quadratic equation $x(x - 1) = 3$ into general form.

$$\begin{array}{ll} \text{Solution:} & x(x - 1) = 3 \\ & x^2 - x = 3 \\ & x^2 - x - 3 = 0 \end{array}$$

Therefore, the general form is $x^2 - x - 3 = 0$. (Where: $a = 1$, $b = -1$, $c = -3$)

Example 6

Change the quadratic equation $5 + \frac{1}{x} - \frac{1}{x^2} = 0$ into general form.

$$\begin{array}{ll} \text{Solution:} & 5 + \frac{1}{x} - \frac{1}{x^2} = 0 \\ & x^2(5 + \frac{1}{x} - \frac{1}{x^2}) = x^2(0) \quad \text{Multiply by } x^2 \\ & 5x^2 + x - 1 = 0 \end{array}$$

Therefore, the general form is $5x^2 + x - 1 = 0$. (Where: $a = 5$, $b = 1$, $c = -1$)

NOW DO PRACTICE EXERCISE 13

**Practice Exercise 13**

1. Identify whether each of the following equations is quadratic. Write Yes or No on the space provided at the right of each equation.

a.	$x^2 = x + 5$	_____
b.	$3x - 4 = x + 1$	_____
c.	$x - 2x^2 = 1 + x^5$	_____
d.	$-2x^2 - 6x = 0$	_____
e.	$6x = 2x + 7$	_____
f.	$5x + 5 = 10x^2$	_____
g.	$7 = 8x$	_____
h.	$5x^2 + 4x + 10 = -8x - 3$	_____
i.	$-7x = 9x^2 + 9x + 3$	_____
j.	$-5x = -3x - 2x^9$	_____

2. Change each of the following quadratic equations in general form, then identify the values of a, b and c.

a. $2x^2 = x + 4$

b. $3x = -x^2 + 7$

c. $5x^2 - 5x + 4 = 6x$

d. $-4x^2 + 3x + 1 = x^2 - 10$

e. $6x = 11x^2$

f. $\frac{2}{3}x^2 - 5x = \frac{1}{2}$

g. $(2x + 2)(3x - 4) = 4x - 5$

h. $8x^2 = 5$

i. $\frac{4}{3}x = 4 + x^2$

j. $x^2 = x - 1$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 14: Solving Quadratic Equations by Factorizing



You identified quadratic equations and its properties in the previous lesson.



In this lesson, you will:

- use the zero product law
- solve quadratic equations by factoring.

Quadratic equations can be solved in many ways. One of this ways is factoring. You learnt factoring of quadratic expressions in Unit 3.

Factoring quadratic expressions provides a different method of writing the expression. However, often expressions have limiting values, for example that the area of a squared room to be tiled is 25 m^2 . This could be written as $x^2 = 25$, if x is the side length of the room.

Solving this equation gives $x = \pm\sqrt{25} = \pm 5$, and in practical situation x would obviously equal to 5 metres.

$x^2 = 25$ could be equally written as
Factoring gives

$$\begin{aligned} x^2 - 25 &= 0 \\ (x + 5)(x - 5) &= 0 \end{aligned}$$

Two quantities are multiplied together and the result is 0. When multiplying two things, the only way zero can be obtained is if one or both of the quantities equals 0.

Think about it: If 4 times a number, for example, equals 0, the number has to be 0.

Therefore, if $(x + 5)(x - 5) = 0$, we can say $x + 5 = 0$ or $x - 5 = 0$
 $x = -5$ or $x = 5$

This can be generalized into the **Zero-Product Property** or the **Null Factor Law**.

The Zero Product Property or Null Factor Law

This law simply states that if the product of two numbers is zero, then one or both of the numbers is zero.

If $ab = 0$, then $a = 0$ or $b = 0$ (which includes $a = b = 0$).

This law helps in the solving of non-linear equations. In solving quadratic equations, first you rearrange the equation so that one side is zero, then factorize so that you have two linear factors. From the zero-product or null factor law, the solutions occur when these factors equal to zero.

Example 1

Suppose you want to solve the equation $x^2 + x - 20 = 0$.

You can factor the left side as: $(x + 5)(x - 4) = 0$

The zero product property says:

$$\text{If } (x + 5)(x - 4) = 0, \text{ then } (x + 5) = 0 \text{ or } (x - 4) = 0$$

Now we just solve each linear equation separately..

$$\text{For } (x + 5) = 0 \text{ we get } x = -5$$

$$\text{For } (x - 4) = 0 \text{ we get } x = 4$$

And the **solutions** or **roots** of the quadratic are $x = -5$ and $x = 4$.

Example 2

Solve $x^2 + 3x - 10 = 0$ by factoring.

Solution: This quadratic is already in its general form.

$$\text{You can factor the left side as: } (x + 5)(x - 2) = 0$$

The zero product property says:

$$\text{If } (x + 5)(x - 2) = 0, \text{ then } (x + 5) = 0 \text{ or } (x - 2) = 0$$

Now we just solve each of these.

$$\text{For } (x + 5) = 0 \text{ we get } x = -5$$

$$\text{For } (x - 2) = 0 \text{ we get } x = 2$$

And the **solutions** or **roots** of the quadratic are $x = -5$ and $x = 2$.

Example 3

Solve $\frac{25}{6}x^2 = \frac{8}{3}$ by factoring.

$$\text{Solution: } \frac{25}{6}x^2 = \frac{8}{3}$$

$$\text{Multiply both sides by 6 to clear fractions. } 6\left(\frac{25}{6}x^2\right) = 6\left(\frac{8}{3}\right)$$

$$25x^2 = 16$$

Subtract 16 on both sides.

$$25x^2 - 16 = 16 - 16$$

Equation in general form

$$25x^2 - 16 = 0$$

Factor the left side
(Difference of 2 squares)

$$(5x + 4)(5x - 4) = 0$$

Use the Zero product law

$$5x + 4 = 0;$$

$$5x - 4 = 0$$

Solve each equation:

$$5x = -4$$

$$5x = 4$$

$$x = -\frac{4}{5}$$

$$x = \frac{4}{5}$$

The solutions or roots of the quadratic equation are $x = -\frac{4}{5}$ and $x = \frac{4}{5}$.

Example 4

Solve the quadratic equation $x(6x + 1) = 2$ by factoring.

Solution: $x(6x + 1) = 2$

$$6x^2 + x = 2$$

Distributive property to clear the brackets

$$6x^2 + x - 2 = 0$$

Write in general form

$$(3x + 2)(2x - 1) = 0$$

Factor the left side

$$3x + 2 = 0; 2x - 1 = 0$$

Zero product property

Solve each equation:

$$3x = -2$$

$$2x = 1$$

$$x = -\frac{2}{3}$$

$$x = \frac{1}{2}$$

The solutions or roots of the quadratic equation are $x = -\frac{2}{3}$ and $x = \frac{1}{2}$.

We can now summarize the steps in solving quadratic equations by factoring.

To solve quadratic equations by factoring:

1. Simplify each side of the quadratic equation if needed. This would involve removing grouping symbols, clearing fractions, collecting like terms, etc.
2. Write the quadratic equation in general form, $ax^2 + bx + c = 0$, if needed.
3. Factor the left side of the equation.
4. Use the Zero Product Law.
5. Solve the resulting equations.

Here are some more examples.

Example 5

Solve the quadratic equation $17y - 12y^2 - 6 = 0$.

Solution:

$$17y - 12y^2 - 6 = 0$$

$$-12y^2 + 17y - 6 = 0 \quad \text{Rearrange the terms in order of the power of } x$$

$$12y^2 - 17y + 6 = 0 \quad \text{Divide by } -1.$$

$$(4y - 3)(3y - 2) = 0 \quad \text{Factorize.}$$

$$4y - 3 = 0 \quad \text{or} \quad 3y - 2 = 0 \quad \text{Zero-Product or Null factor Law}$$

$$4y = 3 \quad \text{or} \quad 3y = 2$$

$$y = \frac{3}{4} \quad \text{or} \quad y = \frac{2}{3}$$

The solutions or roots of the quadratic equation are $y = \frac{3}{4}$ and $y = \frac{2}{3}$.

Now let us work out the next example without showing the steps.

Example 6

Solve the quadratic equation $5x^2 - 14x - 3 = 0$ by factoring.

Solution:

$$5x^2 - 14x - 3 = 0$$

$$(5x + 1)(x - 3) = 0$$

$$5x + 1 = 0 ; x - 3 = 0$$

$$5x = -1 ; \quad x = 3$$

$$x = -\frac{1}{5}$$

The solutions or roots of the quadratic equation are $x = -\frac{1}{5}$ and $x = 3$.

Note:

The zero product or null factor law applies only when one side is zero. If $\mathbf{ab} = 5$, it would be wrong to say that either $\mathbf{a}$ or $\mathbf{b}$ must be 5.

NOW DO PRACTICE EXERCISE 14

**Practice Exercise 14**

Solve the following quadratic equations.

1. $(a + 1)(a + 6) = 0$

2. $(2x + 3)(6x - 5) = 0$

3. $a(a + 10) = 0$

4. $x^2 + 8x + 12 = 0$

5. $2x^2 + 9x + 4 = 0$

6. $13y + 9y^2 - 10 = 0$

7. $14 - 5x - x^2 = 0$

8. $100x^2 - 16$

9. The length of a rectangle is 2 cm more than its width. If the area is 35 cm^2 , find the width of the rectangle.

10. If a number is added to its square, the result is 56. What could the number be?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 15: Solving Quadratic Equations by Square Root Method



You have learned to solve quadratic equations by factoring applying the Zero-Product or Null Factor Law.



In this lesson, you will:

- solve quadratic equations by the square root method

You are already familiar with factoring to solve quadratic equations. However, not all quadratic equations can be factored. In this lesson you will use square roots to learn another way to solve quadratic equations and this method will work with all quadratic equations.

One way to solve the quadratic equation $x^2 = 16$ is to subtract 16 from both sides to get one side of the equation equal to 0. $x^2 - 16 = 0$. The expression on the left side can be factored. $(x + 4)(x - 4) = 0$. Using the zero-product or null factor law, you know this means $x + 4 = 0$ or $x - 4 = 0$. So $x = -4$ or 4 .

Another property would let you solve the quadratic equation $x^2 = 16$ more easily.

The Square Root Property

This property states that:

If **A** and **B** are algebraic expressions such that $A^2 = B$, then $A = \sqrt{B}$ or $A = -\sqrt{B}$. This can also be written $A = \pm\sqrt{B}$.

In other words, if you have an expression squared set equal to another expression, the inverse operation to solve it, is to take the square root of both sides. Since both a positive and its opposite squared result in the same number, then you will have two answers, positive or negative square root of B.

Example 1

Solve the quadratic equation $x^2 = 25$ using the square root property.

Solution: $x^2 = 25$

$$x = \pm\sqrt{25}$$

$$x = \pm 5$$

Since one side is simply x^2 you can take the square root of both sides to get x on one side.

Do not forget to use both positive and negative square roots.

There are two solutions to this quadratic equation: $x = 5$ or $x = -5$.

In example 1, you can take the square root of both sides easily because there is only one term on each side. In some equations, you may need to do some work to get the equation in this form. You will find that this involves isolating x^2 .

Example 2

Solve the quadratic equation $5x^2 = 20$ by square root method.

Solution:

Note that this equation is not of the form $A^2 = B$ to begin with. The 5 is not a part of the expression being squared on the left side of the equation. We can easily write it in the form $A^2 = B$ by dividing both sides by 5.

Hence, $5x^2 = 20$

$$\frac{5x^2}{5} = \frac{20}{5} \quad \text{Divide both sides by 5.}$$

$$x^2 = 4 \quad \text{Written in the form } A^2 = B.$$

$$x = \pm\sqrt{4} \quad \text{Square root method}$$

$$x = \sqrt{4} \text{ or } x = -\sqrt{4} \quad \text{Square root of } 4 = 2$$

$$x = 2 \text{ or } x = -2$$

There are two solutions to the quadratic equation: **$x = 2$ and $x = -2$** .

Example 3

Solve $10x^2 + 5 = 85$

Solution: $10x^2 + 5 = 85$

$$10x^2 + 5 - 5 = 85 - 5 \quad \text{Subtract 5 from both sides of the equation to get the } x^2 \text{ term by itself on the left side.}$$

$$10x^2 = 80$$

$$\frac{10x^2}{10} = \frac{80}{10} \quad \text{Divide both sides by 10 to write the equation in the form } A^2 = B.$$

$$x^2 = 8$$

$$x = \pm\sqrt{8} \quad \text{Use Square Root Property}$$

$$x = \pm\sqrt{(4)(2)} \quad \text{Simplify the radical}$$

$$x = \pm 2\sqrt{2}$$

Therefore, the two solutions of the quadratic equation are **$x = 2\sqrt{2}$ and $x = -2\sqrt{2}$** .

Sometimes more than just the x is being squared.

Example 4

Solve $(2x - 1)^2 - 20 = 0$ by using the square root method.

Solution:	$(2x - 1)^2 - 20 = 0$	Add 20 to both sides of the equation to get the x^2 term by itself on the left side.
	$(2x - 1)^2 = 20$	
	$2x - 1 = \pm\sqrt{20}$	Square root property
	$2x - 1 = \pm\sqrt{(4)(5)}$	Simplify the radical
	$2x - 1 = \pm 2\sqrt{5}$	
	$2x - 1 + 1 = \pm 2\sqrt{5} + 1$	Add 1 to both sides to isolate x
	$2x = \pm 2\sqrt{5} + 1$	
	$\frac{2x}{2} = \frac{\pm 2\sqrt{5} + 1}{2}$	Divide both sides by 2 to solve for x .
	$x = \frac{\pm 2\sqrt{5} + 1}{2}$	

Hence the two solutions to the equation are $x = \frac{2\sqrt{5} + 1}{2}$ and $x = \frac{-2\sqrt{5} + 1}{2}$.

Example 5

Solve the quadratic equation $3x^2 + 7 = 55$ using the Square Root Method.

Solution:	$3x^2 + 7 = 55$	
	$3x^2 + 7 - 7 = 55 - 7$	Subtract 7 from both sides
	$3x^2 = 48$	
	$\frac{3x^2}{3} = \frac{48}{3}$	Divide both sides by 3.
	$x^2 = 16$	
	$x = \pm\sqrt{16}$	Use Square root method
	$x = \pm 4$	
	$x = +4$ or $x = -4$	Answers

Example 6

Solve the quadratic equation $-2x^2 + 15 = x^2 - 12$ using the Square Root Method.

Solution:

$$-2x^2 + 15 = x^2 - 12$$

$$-2x^2 + 15 - 15 = x^2 - 12 - 15 \quad \text{Subtract 15 from both sides}$$

$$-2x^2 = x^2 - 27$$

$$-2x^2 - x^2 = x^2 - x^2 - 27 \quad \text{Subtract } x^2 \text{ from both sides}$$

$$-3x^2 = -27$$

$$\frac{-3x^2}{3} = \frac{27}{3}$$

Divide both sides by 3

$$x^2 = 9$$

Written in the form $A^2 = B$

$$x = \pm\sqrt{9}$$

Apply square root method

$$x = \pm 3$$

$$x = 3 \text{ or } x = -3$$

Answer

Note:

After applying the square root method to a quadratic equation you will end up with either one or two linear equations to solve. Often, you will have two linear equations, but if B is equal to zero, then you will only have one since plus or minus 0 is only one number.

Summarizing the steps done on the above examples, we have:

To solve quadratic equations using the square root method:

1. Write the quadratic equation in the form $A^2 = B$ if needed.
2. Apply the square root method.
3. Solve the resulting equation.

NOW DO PRACTICE EXERCISE 15

**Practice Exercise 15**

Solve the following quadratic equations by applying the square root method.

1. $x^2 - 4 = 0$

2. $x^2 - 50 = 0$

Answer: _____

Answer: _____

3. $(x - 5)^2 - 100 = 0$

4. $(x - 2)^2 - 12 = 0$

Answer: _____

Answer: _____

5. $x^2 - 1 = 24$

6. $-5x^2 - 2 = x^2 - 20$

Answer: _____

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 16: Solving Quadratic by Completing the Square



You learnt to solve quadratic equations by applying the square Root method on the previous lesson.



In this lesson, you will:

- enumerate the steps on how to complete the square
- solve quadratic equation by completing the square

Some quadratic equations are simple to solve because they are of the form $A^2 = B$, and then you take the square root of both sides.

An example would be:

$$(x - 4)^2 = 5$$

$$x - 4 = \pm\sqrt{5}$$

$$x = 4 \pm\sqrt{5}$$

$$x = 4 - \sqrt{5} \text{ and } x = 4 + \sqrt{5}$$

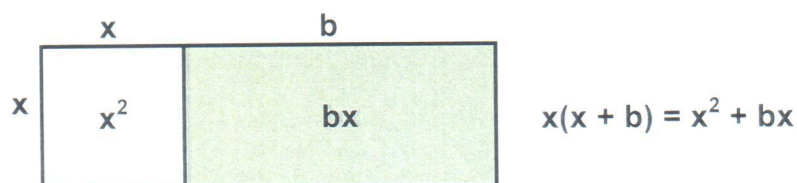
Unfortunately, most quadratic equations do not come neatly squared like this. For your average everyday quadratic, you first have to use the technique of "completing the square" to rearrange the quadratic into the neat "(squared part) equals (a number)" format shown above.

Completing the Square

One way to solve quadratic equations is by **completing the square**. When you do not have a perfect square trinomial, you can create one by adding a constant term that is a perfect square to both sides of the equation.

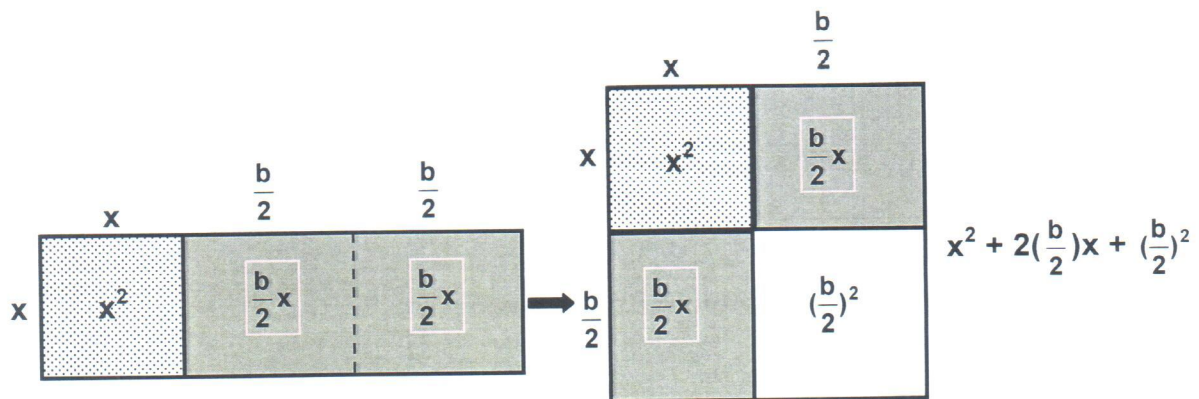
"Completing the Square" does exactly what it says. It takes something that is not a square and makes it one.

See the illustration using an area model of the binomial $x^2 + bx$.



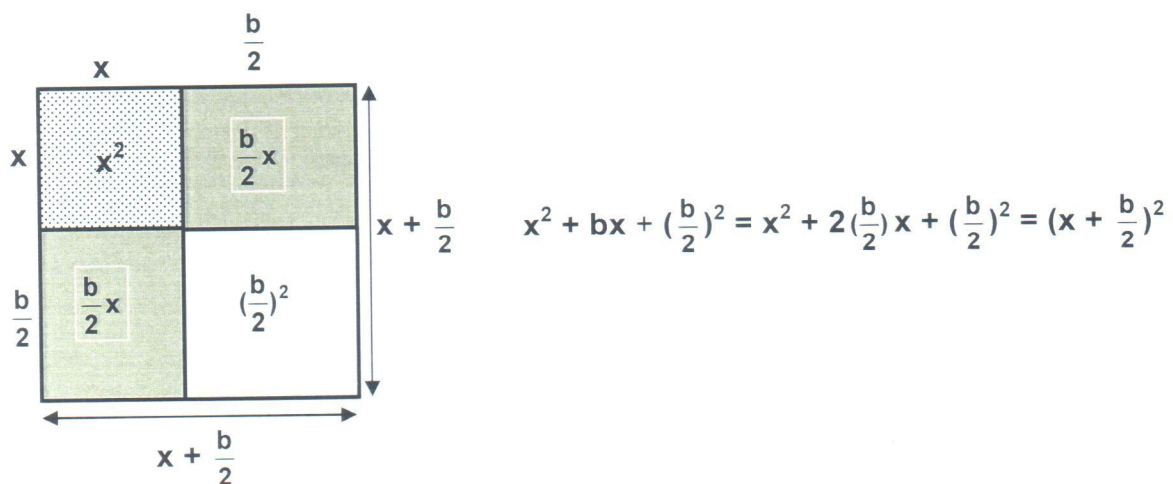
In this example, the area of the whole rectangle is given by $x(x + b)$.

Now let us make the rectangle into a square. First divide the grey rectangle with area bx into two equal rectangles each with area $\frac{b}{2}x$. Then rotate and reposition one of them. The size of the grey area has not changed, it still adds up to bx .



The grey rectangles now make up two sides of a square, shown in white. The area of that square is the length of the grey rectangles squared or $(\frac{b}{2})^2$.

Here comes the interesting part, do you see that when the white square is added to the patterned square and the grey regions, the whole shape is also now a square? In other words, you have completed the square. By adding the quantity $(\frac{b}{2})^2$ to the original binomial, you have made a square, a square with sides of $x + \frac{b}{2}$.



Notice that the area of this square can be written as a squared binomial: $(x + \frac{b}{2})^2$.

To complete the square for an expression of the form $x^2 + bx$:

1. Identify the value of b (the coefficient of the x term).
2. Calculate and add $(\frac{b}{2})^2$

The expression becomes $x^2 + bx + (\frac{b}{2})^2 = (x + \frac{b}{2})^2$

Example 1

Find the number to be added to $x^2 + 8x$ to make it a perfect trinomial square.

Solution:

$$x^2 + 8x$$

Identify b if this has the form $x^2 + bx$.

$$b = 8$$

Complete the square by adding $\left(\frac{b}{2}\right)^2$

$$b = 8 \text{ so } \left(\frac{8}{2}\right)^2 = 4^2 = 16$$

In other words, we complete the square by taking $\frac{1}{2}$ of b (the coefficient of the x term) and then squaring it.

Hence,

$$x^2 + 8x + 16$$

Check that the result is a perfect square trinomial.

$$\begin{aligned}(x + 4)^2 &= x^2 + 4x + 4x + 16 \\ &= x^2 + 8x + 16\end{aligned}$$

So it is a perfect square trinomial.

Therefore, adding **+16** will make $x^2 + 8x$ a perfect square trinomial.

Notice that $\left(\frac{b}{2}\right)^2$ is always positive since it is a square of a number. When you complete the square you are always adding a positive value.

You can use completing the square to solve a quadratic equation that cannot be solved by factoring.

Solving Quadratic Equation using Completing the Square

Let us look at an example where we actually solve quadratic equations using completing the square by finding a value for the variable.

Example 2

$$\text{Solve } x^2 - 12x - 4 = 0$$

$$\text{Solution: } x^2 - 12x - 4 = 0$$

$$x^2 - 12x = 4$$

Rewrite the equation with the left side of the form $x^2 + bx$ to prepare to complete the square. Identify b .

$$b = 12, \text{ so } \left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 6^2 = 36$$

$$x^2 - 12x + 36 = 4 + 36$$

Complete the square by adding 36

$$x^2 - 12x + 36 = 40$$

Simplify

$$(x - 6)^2 = 40$$

Rewrite the left side as a squared binomial

$$x - 6 = \pm \sqrt{40}$$

Use the Square Root method

$$x = 6 \pm \sqrt{40}$$

Solve for x by adding 6 to both sides

$$x = 6 \pm \sqrt{(4)(10)}$$

Simplify the radical

$$x = 6 \pm 2\sqrt{10} \quad \textbf{Answer}$$

You may have noticed that because you have to use square roots, all the examples have two solutions. Here is another example.

Example 2

Solve $x^2 + 16x + 5 = -47$ by completing the square

Solution: $x^2 + 16x + 5 = -47$

$$\begin{aligned} x^2 + 16x &= -64 \\ b &= 16 \end{aligned}$$

Rewrite the equation so the left side has the form $x^2 + bx$. Identify b.

$$x^2 + 16x + 64 = -64 + 64$$

Add $\left(\frac{b}{2}\right)^2$ which is $\left(\frac{16}{2}\right)^2 = 8^2 = 64$ on both sides.

$$(x + 8)^2 = 0$$

Write the left side as a squared binomial.

$$x + 8 = 0$$

Take the square root of both sides.

$$x = -8$$

Solve for x.

Therefore, the solution of the quadratic equation is $x = -8$.

Example 3

Find the x-intercepts of $y = 4x^2 - 2x - 5$.

Solution:

First you have to remember that finding the x-intercepts means setting y equal to zero and solving for the x-values, so this question is really asking you to "Solve $4x^2 - 2x - 5 = 0$ ".

Hence,

$$\begin{aligned} 4x^2 - 2x - 5 &= 0 \\ 4x^2 - 2x &= 5 \end{aligned}$$

Rewrite the equation so that the left side has the form $x^2 + bx$.

$$\begin{aligned} \frac{4x^2}{4} - \frac{2x}{4} &= \frac{5}{4} \\ x^2 - \frac{1}{2}x &= \frac{5}{4} \end{aligned}$$

Divide both sides by 4 to make 1 as the coefficient of x^2 .

$$b = -\frac{1}{2}$$

Identify b.

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{5}{4} + \frac{1}{16}$$

Add $\left(\frac{b}{2}\right)^2$ which is $\left(-\frac{1}{4}\right)^2 = \frac{1}{16}$ on both sides.

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{21}{16}$$

Simplify.

$$\left(x - \frac{1}{4}\right)^2 = \frac{21}{16}$$

Write the left side as a squared term.

$$x - \frac{1}{4} = \pm \sqrt{\frac{21}{16}}$$

Apply square root method.

$$x - \frac{1}{4} = \pm \frac{\sqrt{21}}{4}$$

Simplify the radical.

$$x = \frac{1}{4} \pm \frac{\sqrt{21}}{4}$$

Solve for x.

Therefore, the two solutions are $x = \frac{1}{4} - \frac{\sqrt{21}}{4}$ and $x = \frac{1}{4} + \frac{\sqrt{21}}{4}$.

Now let us try to work out some examples with writing the steps.

Example 4

Solve $x^2 + 6x - 7 = 0$ by completing the square.

Solution:

$$\begin{aligned} x^2 + 6x - 7 &= 0 \\ x^2 + 6x &= 7 \\ x^2 + 6x + 9 &= 7 + 9 \\ x^2 + 6x + 9 &= 16 \\ (x + 3)^2 &= 16 \\ x + 3 &= \pm \sqrt{16} \\ x + 3 &= \pm 4 \\ x &= -3 \pm 4 \\ x &= -3 + 4 \quad \text{or} \quad x = -3 - 4 \\ &= 1 \qquad \qquad \qquad = -7 \end{aligned}$$

Therefore the two solutions are $x = 1$ and $x = -7$.

NOW DO PRACTICE EXERCISE 16

**Practice Exercise 16**

Solve the following quadratic equations by completing the square.

1. $x^2 + 6x + 5 = 0$

2. $x^2 - 2x - 8 = 0$

Answer: _____

Answer: _____

3. $y^2 = 10 - 3y$

4. $m^2 + 16m + 50 = 0$

Answer: _____

Answer: _____

5. $2x^2 + 3x = 2$

6. $x^2 + x = x + 10$

Answer: _____

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 17: Solving Quadratic Equations by Using the Quadratic Formula



You learnt to solve quadratic equations by completing the square in the last lesson.



In this lesson, you will:

- enumerate the steps in solving quadratic equations by the quadratic formula.
- solve quadratic equations using the quadratic formula.

You learnt in Lesson 16 that the method of completing the square can be used to solve any quadratic equations. Now let us use it to solve the general form of the quadratic equation and in this way derive the **quadratic formula**.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = 0 - c$$

$$\frac{ax^2}{a} + \frac{bx}{a} = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a} = \frac{b^2}{4a} - \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

General form of quadratic equation

Subtract **c** from both sides

Divide both sides by **a**.

Take $\frac{1}{2} \left(\frac{b}{a}\right) = \frac{b}{2a}$ then $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a}$

Add $\frac{b^2}{4a}$ to both sides to make the left side a trinomial square.

Add the fractions on the right side

Factor the left side

Used square root method

Simplify radicals

Subtract $\frac{b}{2a}$ from both sides

Quadratic Formula

When a quadratic equation does not factor or is difficult to factor, you can solve the equation by using the **quadratic formula**.

The procedure for using the quadratic formula can be summarized as follows:

To solve quadratic equations by formula

1. Arrange the equation in general form: $ax^2 + bx + c = 0$
2. Identify the values of a, b, and c.
3. Substitute the values of a, b, and c in the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a \neq 0)$$

4. Simplify your answer.
5. Check your answer by substituting in them original equation.

Example 1

Solve $2x^2 - 5x + 1 = 0$ by using the quadratic formula.

Solution: $2x^2 - 5x + 1 = 0$

This quadratic equation is already arrange in general form.

$a = 2, b = -5, c = 1$

Identify a, b, and c.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

Substitute these values in the formula.

$$x = \frac{5 \pm \sqrt{25 - 8}}{4}$$

Simplify.

$$x = \frac{5 \pm \sqrt{17}}{4}$$

Answer

Example 2

Solve $x^2 + 9 = 6x$ by using the quadratic formula.

Solution: $x^2 + 9 = 6x$

$$x^2 - 6x + 9 = 0$$

Arrange in general form

$a = 1, b = -6, c = 9$

Identify a, b, and c

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

Substitute these values in the formula.

$$x = \frac{6 \pm \sqrt{36 - 36}}{2}$$

Simplify.

$$x = \frac{6 \pm \sqrt{0}}{2}$$

$$x = \frac{6}{2}$$

$$x = 3$$

Answer

Example 3

Solve $x^2 + 4x - 5 = 0$ by quadratic formula.

Solution:

$$x^2 + 4x - 5 = 0$$

Arrange in general form

$$a = 1, b = 4, c = -5$$

Identify a, b, and c

$$x = \frac{- (4) \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$

Substitute these values in the formula.

$$x = \frac{-4 \pm \sqrt{16 + 20}}{2}$$

Simplify.

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

$$x = \frac{-4 \pm 6}{2}$$

$$x = \frac{2}{2} = 1 \text{ or } x = \frac{-10}{2} = -5$$

The roots of the equations are $x = 1$ and $x = -5$.

Example 4

Solve $x^2 - 6x + 8 = 0$ using quadratic formula.

Solution: $x^2 - 6x + 8 = 0$

General form

$$a = 1, b = -6, c = 8$$

identify a, b, and c.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

Substitute these values in the formula.

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

Simplify.

$$x = \frac{6 \pm \sqrt{4}}{2}$$

$$x = \frac{6 \pm 2}{2}$$

$$x = \frac{6+2}{2} ; x = \frac{6-2}{2}$$

$$x = \frac{8}{2} = 4 ; x = \frac{4}{2} = 2$$

The solutions are $x = 4$ and $x = 2$.

Example 5

Solve $3x^2 = 4x + 1$ by the quadratic formula.

Solution: $3x^2 = 4x + 1$

$$3x^2 - 4x - 1 = 0$$

General form.

$$a = 3, b = -4, c = -1$$

Identify a, b, and c.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$

Substitute these values in the formula.

$$x = \frac{4 \pm \sqrt{16 + 12}}{6}$$

Simplify.

$$x = \frac{4 \pm \sqrt{28}}{6}$$

$$x = \frac{4 \pm \sqrt{(4)(7)}}{6}$$

Simplify the radical.

$$x = \frac{4 \pm 2\sqrt{7}}{6}$$

$$x = \frac{2(2 \pm \sqrt{7})}{2(3)}$$

Factor out 2 and cancel.

$$x = \frac{2 \pm \sqrt{7}}{3}$$

This is simplified.

The solutions are $x = \frac{2 + \sqrt{7}}{3}$ and $x = \frac{2 - \sqrt{7}}{3}$.

In general, to solve quadratic equations, the methods of factoring and extracting the square roots, when applicable, are the easiest and quickest to apply. If these two methods are not appropriate, it is best to go directly to the quadratic formula rather than to method of completing the square.

NOW DO PRACTICE EXERCISE 17

**Practice Exercise 17**

Solve the following quadratic equations by using the quadratic formula.

1. $x^2 - 8x + 15 = 0$

2. $x^2 - 25 = 0$

Answer: _____

Answer: _____

3. $3y^2 + 10y = -3y - 4$

4. $4x(x + 3) = 9x$

Answer: _____

Answer: _____

5. $5x^2 + x = 1$

6. $6x^2 - 9x - 12 = 0$

Answer: _____

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 18: Graphs of Quadratic Equations



In the previous lessons you learnt to work out the roots of quadratic equations.



In this lesson, you will:

- identify the characteristics or features of a parabola
- enumerate the steps in graphing the different forms quadratic equations
- sketch the graph of the different forms of quadratic equations such as: $y = x^2$, $y = x^2 + c$ and $y = ax^2 + bx + c$.

In earlier lessons we have shown you how to solve quadratic equations by factoring. A quadratic equation as you remember is an equation that can be written on the standard form

$$ax^2 + bx + c = 0, \text{ where } a \neq 0$$

You know by now how to solve a quadratic equation using factoring, completing the square, and by quadratic formula. Another way of solving a quadratic equation is to solve it graphically.

When graphed, quadratic equations give a smooth U-shaped or a reversed U-shaped curve called a **parabola**. Graphing quadratic equation is a matter of finding the vertex, direction, and often its x- and y-intercepts. In case of relatively simple quadratic equations, using table of values for x and y and plotting the curve based on the resulting points may also be enough.

Let us start with the simplest form of quadratic equation.

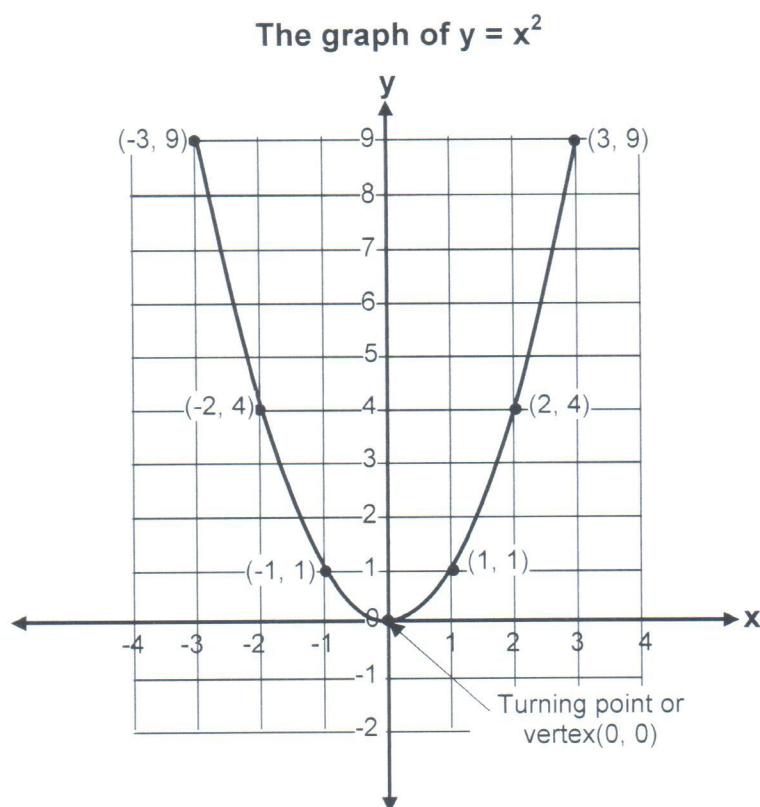
The Quadratic equation of the form $y = x^2$

The equation $y = x^2$ is a quadratic equation.

Let us substitute any values for x and solve for y and plot the corresponding coordinates and graph the equation $y = x^2$ using the table of values below.

x	$y = x^2$	(x, y)
-3	$(-3)^2 = 9$	(-3, 9)
-2	$(-2)^2 = 4$	(-2, 4)
-1	$(-1)^2 = 1$	(-1, 1)
0	0	(0, 0)
1	$(1)^2 = 1$	(1, 1)
2	$(2)^2 = 4$	(2, 4)
3	$(3)^2 = 9$	(3, 9)

Let us graph the equation.



Features of the parabola $y = x^2$

- The graph is symmetrical about the y-axis.
- The **axis of symmetry** is also called the axis of the parabola. For $y = x^2$, the equation of the axis of symmetry is $x = 0$ (the equation of the y-axis).
- The point of intersection of a parabola with its axis (or with the axis of symmetry) is the **vertex**. For $y = x^2$, the vertex is at point **(0, 0)**. The vertex is also called the **turning point**, since the parabola changes direction at this point.
- The y-intercept is the point where the parabola cuts the y-axis. For $y = x^2$, the **y-intercept** is **0**.
- The **x-intercept** is **0**.
- The **minimum value** (Lowest point) of $y = x^2$ is **0**.
- The parabola is **concave up** (opens up) U-shape. 

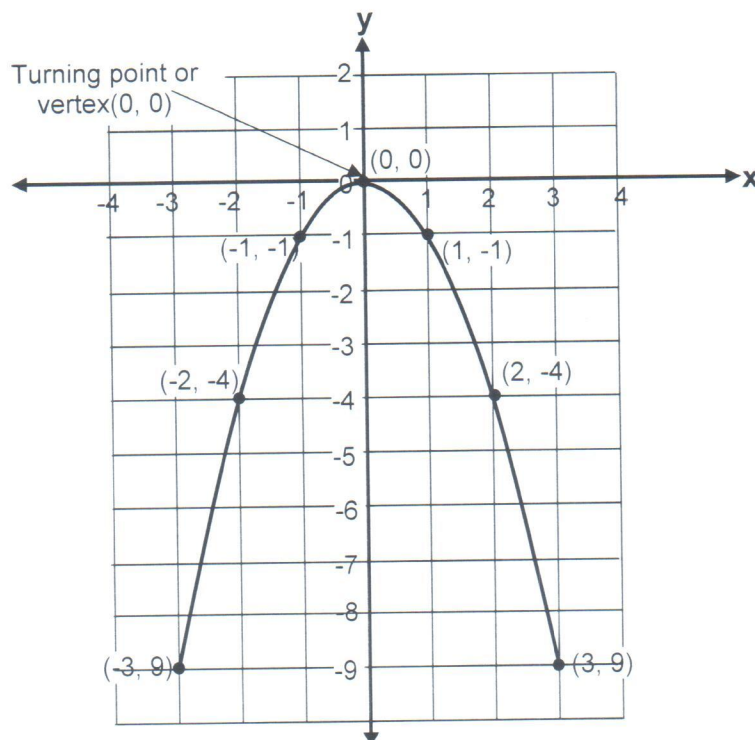
An inverted (turned upside down) parabola is obtained by graphing the quadratic equation $y = -x^2$.

Let us make a table of values and plot the points and then draw the graph of the equation $y = -x^2$.

Table of values for $y = -x^2$.

x	$y = -x^2$	(x, y)
-3	$-(-3)^2 = -9$	(-3, -9)
-2	$-(-2)^2 = -4$	(-2, -4)
-1	$-(-1)^2 = -1$	(-1, -1)
0	0	(0, 0)
1	$-(1)^2 = -1$	(1, -1)
2	$-(2)^2 = -4$	(2, -4)
3	$-(3)^2 = -9$	(3, -9)

The graph of $y = -x^2$

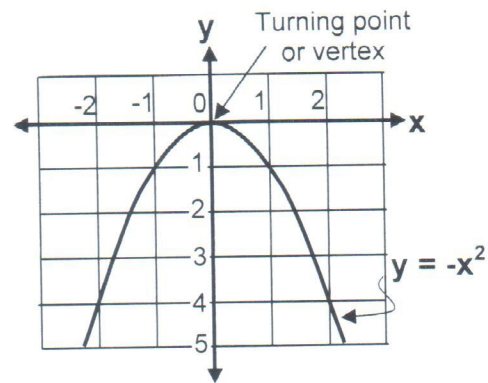
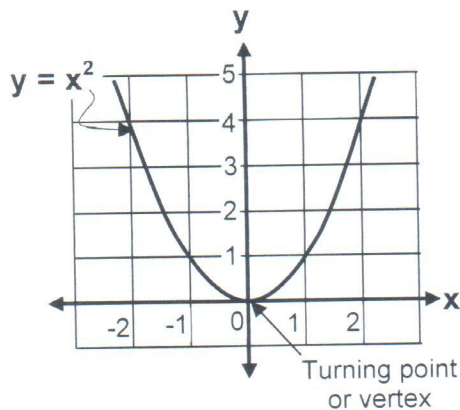


Features of the parabola $y = -x^2$

- The graph is symmetrical about the y-axis.
- The **axis of symmetry** is also called the axis of the parabola. For $y = -x^2$, the equation of the axis of symmetry is $x = 0$ (the equation of the y-axis).
- The point of intersection of a parabola with its axis (or with the axis of symmetry) is the **vertex**. For $y = -x^2$, the vertex is at point **(0, 0)**. The vertex is also called the **turning point**, since the parabola changes direction at this point.
- The y-intercept is the point where the parabola cuts the y-axis. For $y = -x^2$, the **y-intercept** is 0.
- The **x-intercept** is 0.
- The **maximum value** (Highest point) of $y = -x^2$ is 0.
- The parabola is **concave down** (Opens down) U-shape. ----->



The parabola $y = x^2$ is **concave up**, and so it has a **minimum turning point**. The parabola $y = -x^2$ is **concave down**, and so it has a **maximum turning point**.



In general, if the coefficient of the x^2 term of a quadratic is **positive**, the parabola **opens upward**, if the coefficient of the x^2 term of a quadratic is **negative**, the parabola **opens downward**.

Example 1

Let us investigate the graphs of $y = 2x^2$, $y = -2x^2$, $y = 3x^2$ and $y = -3x^2$. What do they look like?

Solution:

Using table of values for $y = \pm 2x^2$ and $y = \pm 3x^2$ and comparing their graphs with the graphs of $y = \pm x^2$ can help us answer the question.

Table of values for $y = x^2$

x	$y = x^2$	(x, y)
-3	$(-3)^2 = 9$	$(-3, 9)$
-2	$(-2)^2 = 4$	$(-2, 4)$
-1	$(-1)^2 = 1$	$(-1, 1)$
0	$(0)^2 = 0$	$(0, 0)$
1	$(1)^2 = 1$	$(1, 1)$
2	$(2)^2 = 4$	$(2, 4)$
3	$(3)^2 = 9$	$(3, 9)$

Table of values for $y = -x^2$

x	$y = -x^2$	(x, y)
-3	$-(-3)^2 = -9$	$(-3, -9)$
-2	$-(-2)^2 = -4$	$(-2, -4)$
-1	$-(-1)^2 = -1$	$(-1, -1)$
0	$-(0)^2 = 0$	$(0, 0)$
1	$-(1)^2 = -1$	$(1, -1)$
2	$-(2)^2 = -4$	$(2, -4)$
3	$-(3)^2 = -9$	$(3, -9)$

Table of values for $y = 2x^2$

x	$y = 2x^2$	(x, y)
-3	$2(-3)^2 = 2(9) = 18$	$(-3, 18)$
-2	$2(-2)^2 = 2(4) = 8$	$(-2, 8)$
-1	$2(-1)^2 = 2(1) = 2$	$(-1, 2)$
0	$2(0)^2 = 0$	$(0, 0)$
1	$2(1)^2 = 2(1) = 2$	$(1, 2)$
2	$2(2)^2 = 2(4) = 8$	$(2, 8)$
3	$2(3)^2 = 2(9) = 18$	$(3, 18)$

Table of values for $y = -2x^2$

x	$y = -2x^2$	(x, y)
-3	$-2(-3)^2 = -2(9) = -18$	$(-3, -18)$
-2	$-2(-2)^2 = -2(4) = -8$	$(-2, -8)$
-1	$-2(-1)^2 = -2(1) = -2$	$(-1, -2)$
0	$-2(0)^2 = 0$	$(0, 0)$
1	$-2(1)^2 = -2(1) = -2$	$(1, -2)$
2	$-2(2)^2 = -2(4) = -8$	$(2, -8)$
3	$-2(3)^2 = -2(9) = -18$	$(3, -18)$

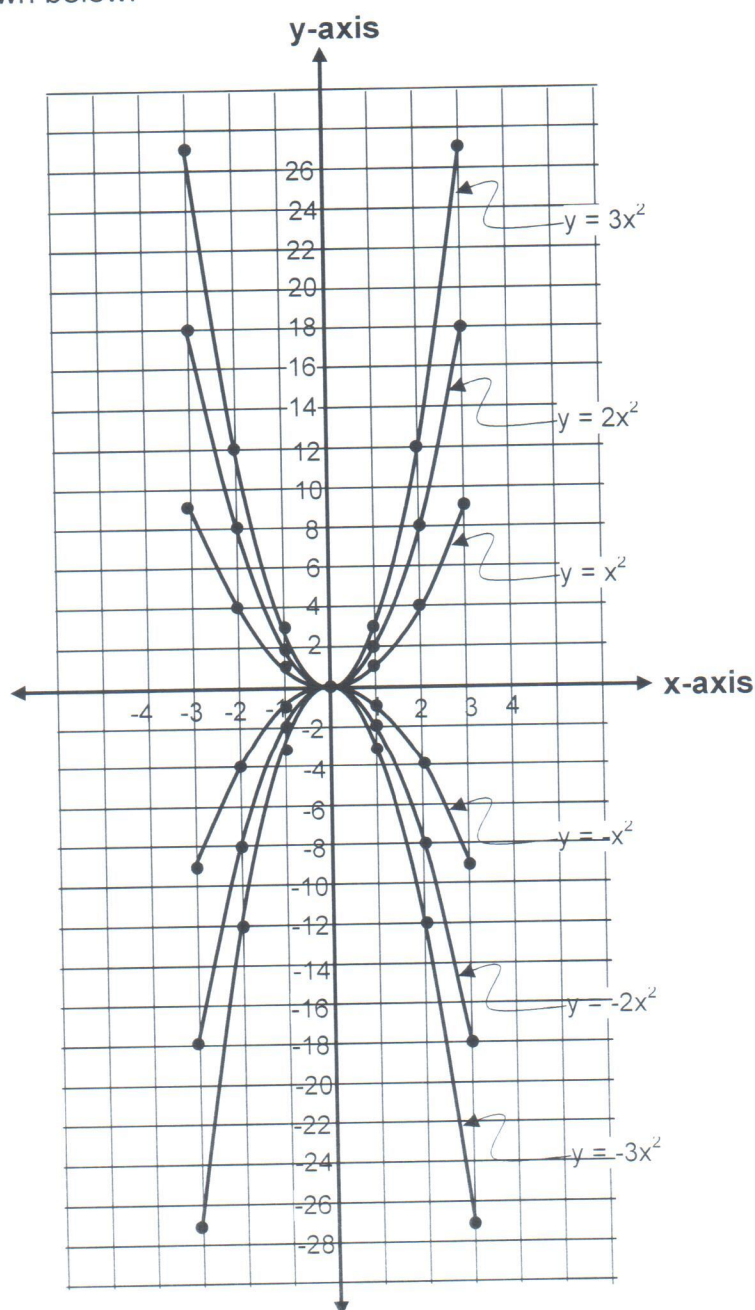
Table of values for $y = 3x^2$

x	$y = 3x^2$	(x, y)
-3	$3(-3)^2 = 3(9) = 27$	(-3, 27)
-2	$3(-2)^2 = 3(4) = 12$	(-2, 12)
-1	$3(-1)^2 = 3(1) = 3$	(-1, 3)
0	$3(0)^2 = 0$	(0, 0)
1	$3(1)^2 = 3(1) = 3$	(1, 3)
2	$3(2)^2 = 3(4) = 12$	(2, 12)
3	$3(3)^2 = 3(9) = 27$	(3, 27)

Table of values for $y = -3x^2$

x	$y = -3x^2$	(x, y)
-3	$-3(-3)^2 = -2(9) = -27$	(-3, -27)
-2	$-3(-2)^2 = -2(4) = -12$	(-2, -12)
-1	$-3(-1)^2 = -2(1) = -3$	(-1, -3)
0	$-3(0)^2 = 0$	(0, 0)
1	$-3(1)^2 = -2(1) = -3$	(1, -3)
2	$-3(2)^2 = -2(4) = -12$	(2, -12)
3	$-3(3)^2 = -2(9) = -27$	(3, -27)

The graphs are shown below.



The graphs shows that all of the y- values for $y = \pm 2x^2$ and $y = \pm 3x^2$ are twice and thrice those for $y = \pm x^2$ respectively.

Looking at the graph, you can see that all the graphs are parabolas with vertex at the origin, $(0, 0)$. Notice also that as the coefficient of the x^2 term **increases**, the graph stretches (dilates) vertically.

In general, the coefficient of the x^2 term of a quadratic equation determines whether the parabola is **wide** or **narrow**.

The turning point of the parabola is not always at the origin of a graph. The position of the turning point depends on the equation of the parabola.

Consider the equations $y = x^2$, $y = x^2 - 4$ and $y = x^2 + 4$.

Let us make the table of values for each quadratic equation using the standard values for x we used in the previous examples. $(-3, -2, -1, 0, 1, 2, 3)$.

For $y = x^2$

x	$y = x^2$	(x, y)
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$

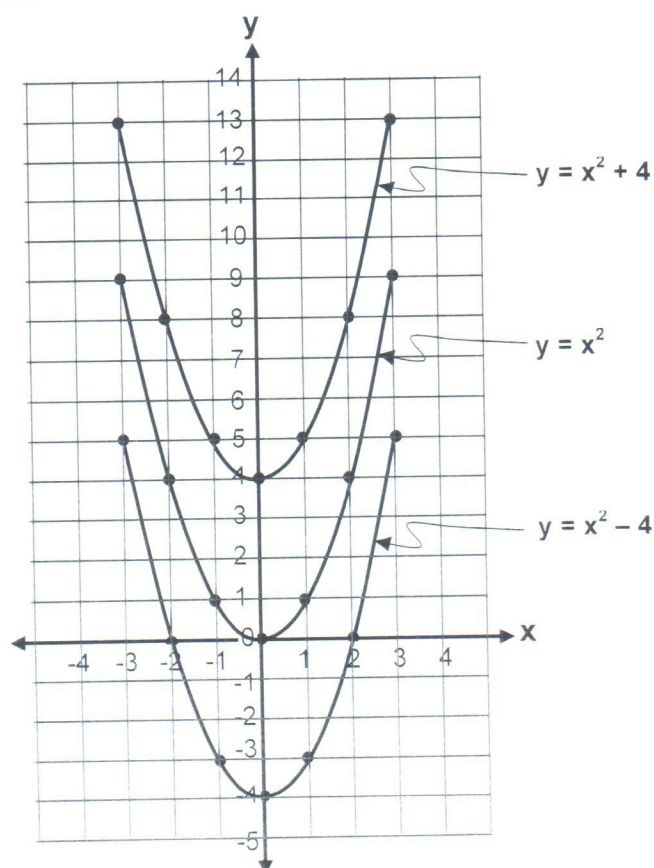
For $y = x^2 + 4$

x	$y = x^2 + 4$	(x, y)
-3	$(-3)^2 + 4 = 13$	$(-3, 13)$
-2	$(-2)^2 + 4 = 8$	$(-2, 8)$
-1	$(-1)^2 + 4 = 5$	$(-1, 5)$
0	$(0)^2 + 4 = 4$	$(0, 4)$
1	$(1)^2 + 4 = 5$	$(1, 5)$
2	$(2)^2 + 4 = 8$	$(2, 8)$
3	$(3)^2 + 4 = 13$	$(3, 13)$

For $y = x^2 - 4$

x	$y = x^2 - 4$	(x, y)
-3	$(-3)^2 - 4 = 5$	$(-3, 5)$
-2	$(-2)^2 - 4 = 0$	$(-2, 0)$
-1	$(-1)^2 - 4 = -3$	$(-1, -3)$
0	$(0)^2 - 4 = -4$	$(0, -4)$
1	$(1)^2 - 4 = -3$	$(1, -3)$
2	$(2)^2 - 4 = 0$	$(2, 0)$
3	$(3)^2 - 4 = 5$	$(3, 5)$

We will plot all the three graphs using the same axes. Notice again that the points are joined by a smooth curve.



You can see that every point on the graphs of $y = x^2 + 4$ and $y = x^2 - 4$ is 4 units higher than the corresponding point on $y = x^2$.

The coordinates of the turning point for the graph of $y = x^2$ are $(0, 0)$, the coordinates for the turning point for the graph of $y = x^2 + 4$ are $(0, 4)$ and the coordinates for the turning point for the graph of $y = x^2 - 4$ are $(0, -4)$.

The **line of symmetry** for the three graphs is $x = 0$ (the y-axis).

The point where the parabola crosses the y-axis is called the **y-intercept**.

The curves $y = x^2$, $y = x^2 + 4$ and $y = x^2 - 4$ cross the y-axis at the points where $x = 0$, $y = 0$; $x = 0$, $y = 4$ and $x = 0$, $y = -4$ respectively.

Notice that the y-intercept is equal to the value of y obtained by setting $x = 0$ into the equation the equation of the parabola in each case.

So for the quadratic equation $y = x^2$, the y-intercept $= 0^2 = 0$; for $y = x^2 + 4$, the y-intercept $= 0^2 + 4 = 4$; and for $y = x^2 - 4$, the y-intercept $= 0^2 - 4 = -4$.

Both the $y = x^2 + 4$ and $y = x^2 - 4$ are quadratic equations of the form $y = x^2 + c$.

The graph of the quadratic equation of the form $y = x^2 + c$ is the standard parabola $y = x^2$ translated vertically by c units. If c is **positive**, the graph moves **upward**. If c is **negative**, the graph moves **downward**.

This means that changing the value of c only changes the vertical position of the graph, not its shape.

Example 1

Sketch the graphs of $y = x^2 - 1$ and $y = x^2 + 0.5$ using the same axes.

Solution:

First we make the table of values for the two equations by substituting any values for x and solve for y and plot the corresponding coordinates.

Table of values for $y = x^2 - 1$

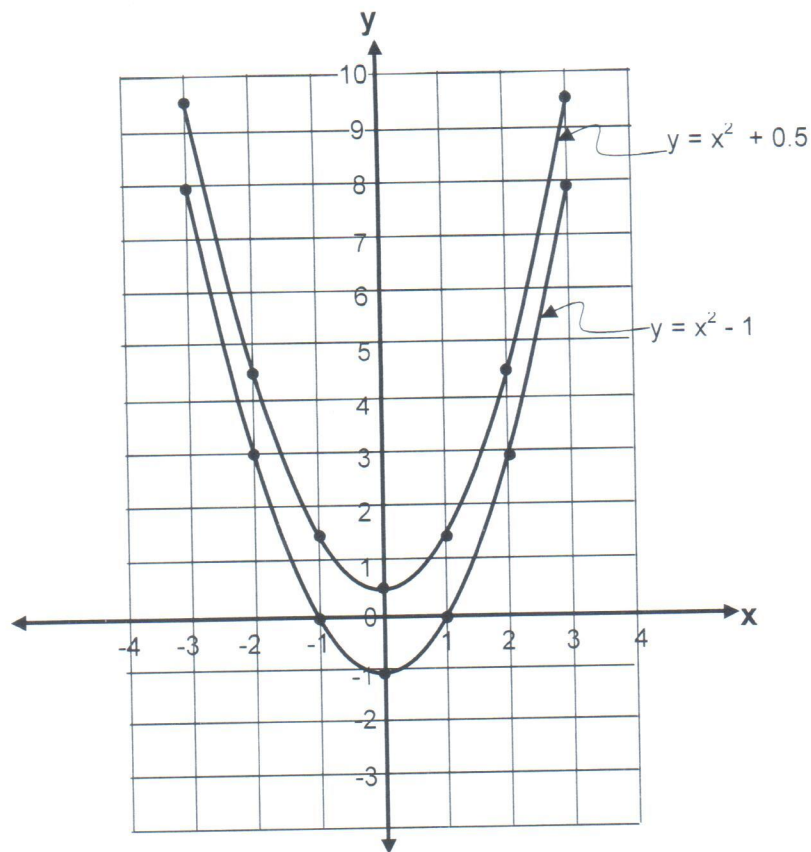
x	$y = x^2 - 1$	(x, y)
-3	$(-3)^2 - 1 = 8$	$(-3, 8)$
-2	$(-2)^2 - 1 = 3$	$(-2, 3)$
-1	$(-1)^2 - 1 = 0$	$(-1, 0)$
0	$(0)^2 - 1 = -1$	$(0, -1)$
1	$(1)^2 - 1 = 0$	$(1, 0)$
2	$(2)^2 - 1 = 3$	$(2, 3)$
3	$(3)^2 - 1 = 8$	$(3, 8)$

Table of values for $y = x^2 + 0.5$

x	$y = x^2 + 0.5$	(x, y)
-3	$(-3)^2 + 0.5 = 9.5$	$(-3, 9.5)$
-2	$(-2)^2 + 0.5 = 4.5$	$(-2, 4.5)$
-1	$(-1)^2 + 0.5 = 1.5$	$(-1, 1.5)$
0	$(0)^2 + 0.5 = 0.5$	$(0, 0.5)$
1	$(1)^2 + 0.5 = 1.5$	$(1, 1.5)$
2	$(2)^2 + 0.5 = 4.5$	$(2, 4.5)$
3	$(3)^2 + 0.5 = 9.5$	$(3, 9.5)$

Since the coefficient of the x^2 term is positive, the parabolas open upward.

Now draw the graphs by joining the points by a smooth curve.



The coordinates of the turning point for the graph of $y = x^2 - 1$ are **(0, -1)** and the coordinates of the turning point for the graph of $y = x^2 + 0.5$ are **(0, 0.5)**.

The **line of symmetry** for the two graphs is **$x = 0$** (the y- axis).

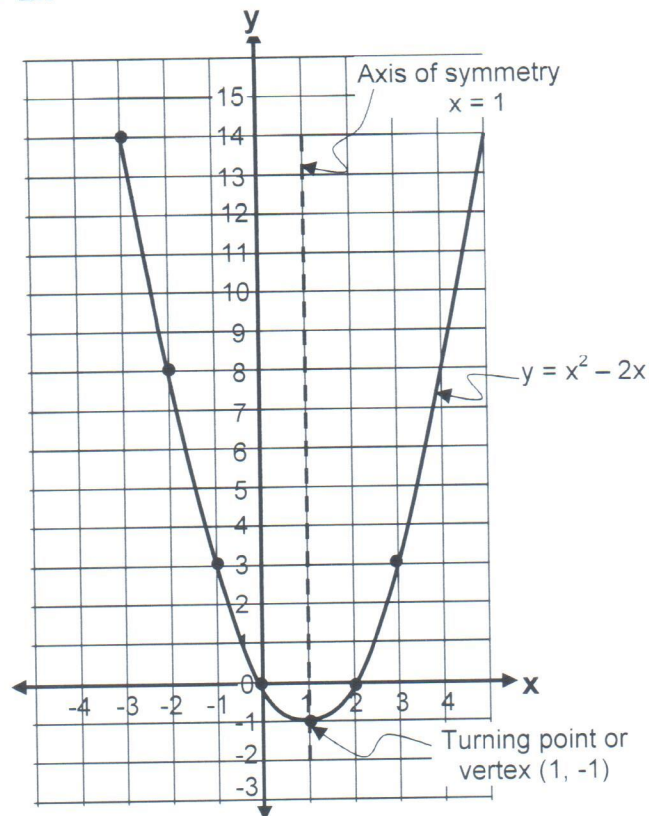
Parabolas in the standard form $y = ax^2 + bx + c$, where $c = 0$

Let us try graphing another parabola where $a = 1$, $b = -2$ and $c = 0$. So we would have the equation $y = x^2 - 2x$. Let us substitute the same values for x as we did in the previous tables and see what we get for y .

Table of values

x	$y = x^2$	$y = x^2 - 2x$	(x, y)
-3	9	$9 - 2(-3) = 9 + 6 = 15$	$(-3, 15)$
-2	4	$4 - 2(-2) = 4 + 4 = 8$	$(-2, 8)$
-1	1	$1 - 2(-1) = 1 + 2 = 3$	$(-1, 3)$
0	0	$0 - 2(0) = 0$	$(0, 0)$
1	1	$1 - 2(1) = 1 - 2 = -1$	$(1, -1)$
2	4	$4 - 2(2) = 4 - 4 = 0$	$(2, 0)$
3	9	$9 - 2(3) = 9 - 6 = 3$	$(3, 3)$

The graph of $y = x^2 - 2x$



Here, you can see again that the x- and y-intercepts are both (0, 0) as the parabola crosses through the origin. The lowest point on the graph is (1, -1) and this is called the **turning point** or **vertex**. If we draw a vertical line through the vertex, the parabola is split into halves so that either side of the vertical line is symmetrical with each other. We call this vertical line the **line or axis of symmetry**. Remember from earlier lesson that vertical lines are always in the form $x = c$ and to graph vertical line, we go across the x-axis to the value of c . In this case, the line or axis of symmetry is the equation $x = 1$.

Parabolas in the standard form $y = ax^2 + bx + c$, where a , b , and $c \neq 0$

Features of the parabola $y = ax^2 + bx + c$

- If $a > 0$, the parabola opens upward and the vertex yields the minimum value or lowest point of the parabola. If $a < 0$, the parabola opens downward and the vertex yields the maximum value or highest point of the parabola.

- The x-intercepts of the parabola are found by solving the equation:

$$ax^2 + bx + c = 0.$$

- The x- coordinate of the vertex may be found using the equation $x = -\frac{b}{2a}$.

The y-coordinate of the vertex may then be found by substituting the x-value into the original equation.

- The axis of symmetry is $x = -\frac{b}{2a}$.

Example 1

Sketch the graph of the quadratic equation $x^2 + 6x + 5 = 0$.

Solution:

Write in standard form $y = x^2 + 6x + 5$

Since $a = 1 > 0$, the parabola opens upward. The vertex is a lowest point.

With $a = 1$ and $b = 6$, the x-coordinate of the vertex $= -\frac{b}{2a} = -\frac{6}{2(1)} = -\frac{6}{2} = -3$

Substitute $x = -3$ in the equation $y = x^2 + 6x + 5$ to find the y-coordinate

$$\begin{aligned} y &= (-3)^2 + 6(-3) + 5 \\ y &= 9 - 18 + 5 \\ y &= -4 \end{aligned}$$

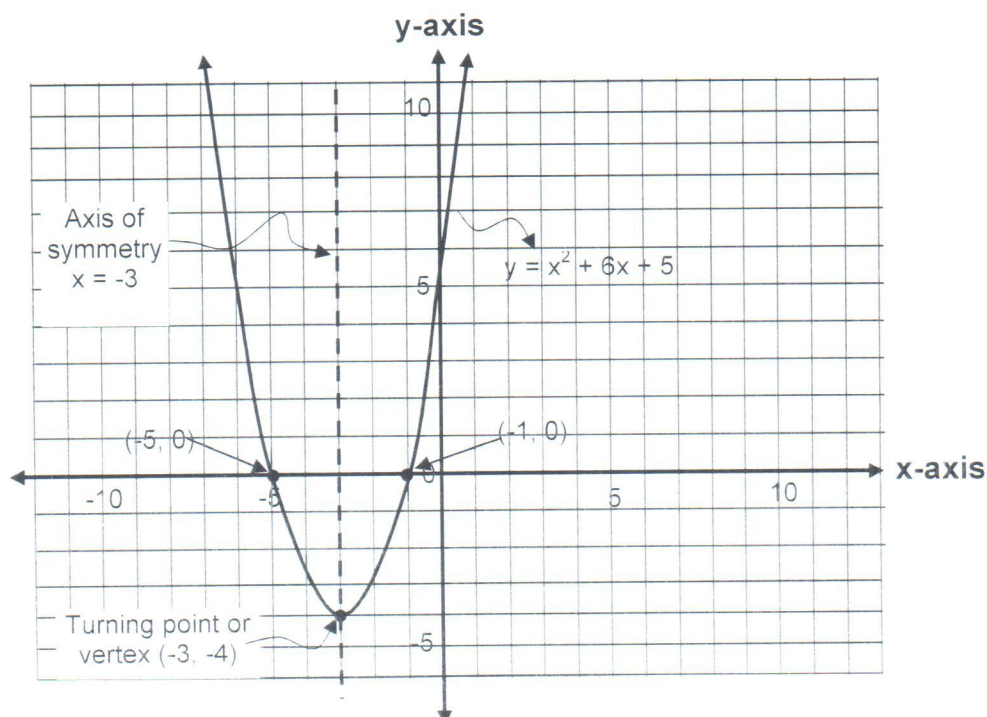
So the vertex is **$(-3, -4)$** .

So, the axis of symmetry is $x = -\frac{6}{2}$ or **$x = -3$**

With the vertex located below the x-axis, and the parabola opens up, we look for the x-intercepts by evaluating the equation:

$$\begin{aligned} x^2 + 6x + 5 &= 0 \\ (x + 5)(x + 1) &= 0 \\ x + 5 = 0 \quad \text{or} \quad x + 1 &= 0 \\ x = -5 \quad \text{or} \quad x &= -1 \end{aligned}$$

Thus, the x-intercepts are $(-5, 0)$ and $(-1, 0)$. The graph is given below.



The parabola cross the y-axis at $y = 5$. This is the **y-intercept**.

The parabola cross the x-axis at $x = -5$ and $x = -1$. These are **the roots** of the quadratic equation.

We can compare this solution to the one we would get if we were to solve the quadratic equation by factoring as we've done earlier.

To graph a quadratic equation $y = ax^2 + bx + c$

1. Determine whether the graph opens up ($a > 0$) or down ($a < 0$).
2. Find the vertex and axis of symmetry using the fact that the x-coordinate of the vertex is $-\frac{b}{2a}$ and the axis of symmetry is $x = -\frac{b}{2a}$.

The y-coordinate of the vertex is found by evaluating substituting the x- value into the original equation.

3. If the vertex is located above the x-axis and the graph opens down, or if the vertex is located below the x-axis and opens up, find the x-intercepts by solving the equation $ax^2 + bx + c = 0$.
4. Plot the vertex and the x-intercepts (or two other points on either side of the line of symmetry if there are no x-intercepts) and sketch the parabola.

NOW DO PRACTICE EXERCISE 17



Practice Exercise 18

1. State the y-intercept value where each the following equations would cross the y-axis:

(a) $y = x^2 + 9$

(b) $y = \frac{1}{2}x^2 - 4$

(c) $y = -x^2$

Answer: _____

Answer: _____

Answer: _____

2. Complete the table of values below for the equation: $y = 3x^2$

x	-3	-2	-1	0	1	2	3
$y = 3x^2$	27						

- (b) Complete the table of values below for the equation: $y = \frac{1}{3}x^2$

x	-9	-6	-3	0	3	6	9
$y = \frac{1}{3}x^2$	27						

- (c) Complete the table of values below for the equation: $y = -10x^2$

x	-4	-3	-2	-1	0	1	2
$y = -10x^2$	-160						

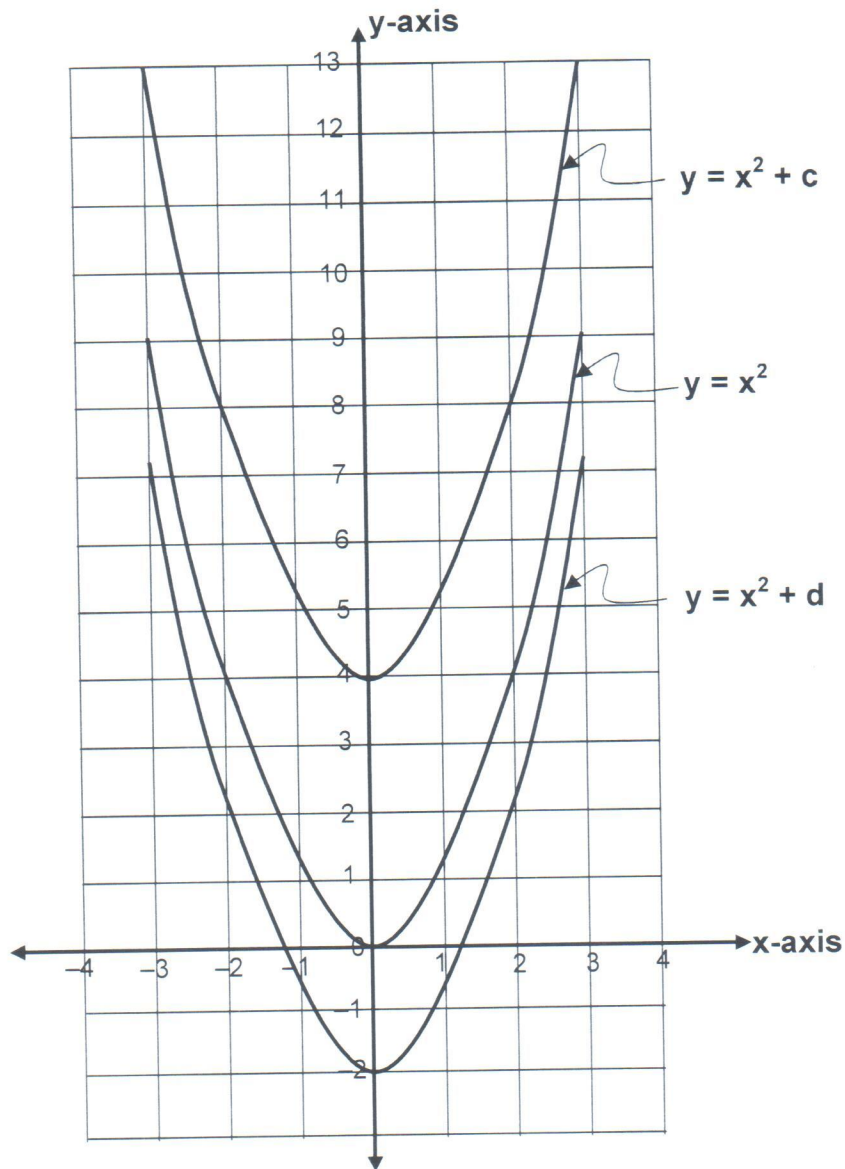
- (d) Complete the table of values below for the equation: $y = 3x^2 - 3$

x	-3	-2	-1	0	1	2	3
$3x^2$	27						
-3	-3						
$y = 3x^2 - 3$	24						

- (e) Complete the table of values below for the equation: $y = x^2 - 2x$

x	-6	-4	-2	0	2	4	6
x^2	36						
-2x	12						
$y = x^2 - 2x$	24						

3. Study the three parabolas shown below.



- (a) Give the coordinates of the turning point for each parabola:

(i) $y = x^2 + d$

(ii) $y = x^2$

(iii) $y = x^2 + c$

Answer: _____ Answer: _____ Answer: _____

- (b) What is the equation of the line of symmetry for each parabola?

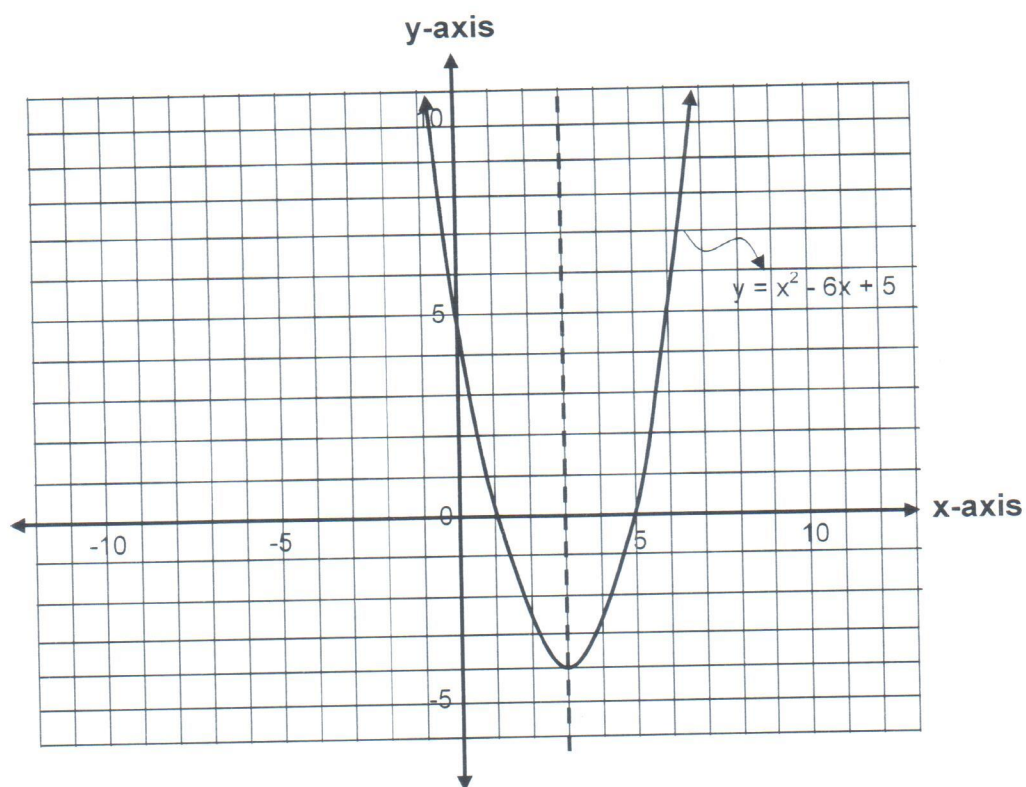
Answer: _____

- (c) What are the values of (i) c and (ii) d in these graphs?

(i) Answer: _____

(ii) Answer: _____

4. Study the parabola shown below.



- a. Give the coordinates of the turning point of the parabola.

Answer: _____

- b. What is the intercept on the y-axis of the parabola?

Answer: _____

- c. What are the x-intercepts of the parabola?

Answer: _____

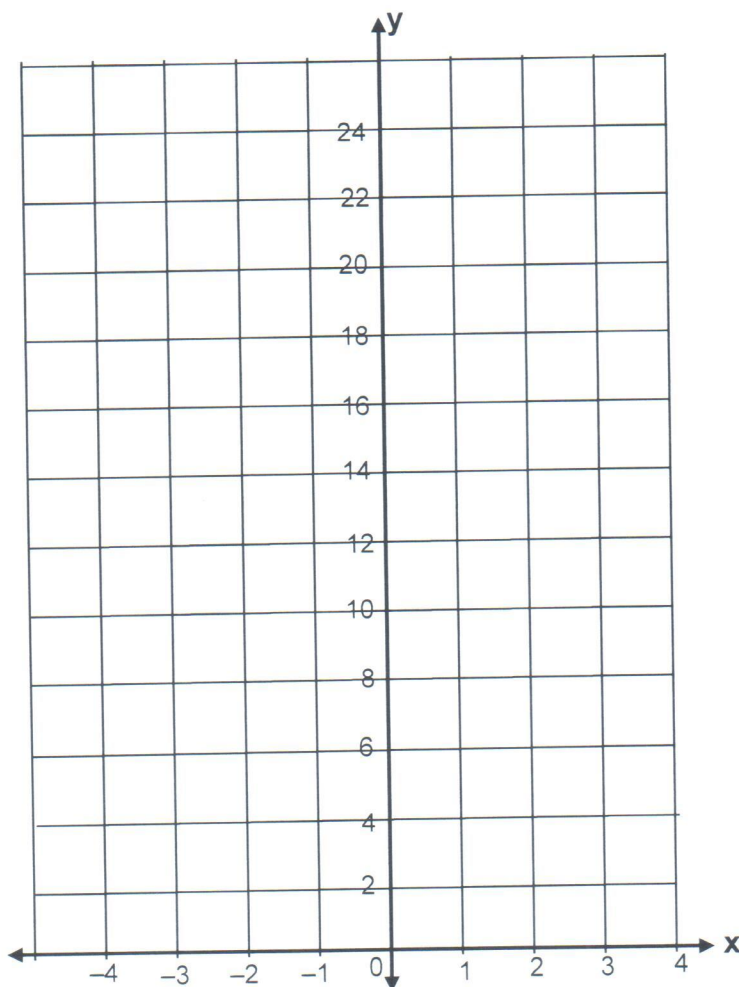
- d. What is the equation of the axis of symmetry?

Answer: _____

5. Draw the graph of $y = 2x^2 + 3$ after completing the table of values shown below.

x	-3	-2	-1	0	1	2	3
$2x^2$							
$+3$							
$y=2x^2+3$							

(a)



- (b) What is the y -intercept?

Answer: _____

- (c) What are the co-ordinates of the turning point?

Answer: _____

- (d) What is the equation of the line of symmetry?

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

TOPIC 3: SUMMARY



This summarizes the important concepts and ideas to be remember

- A **Quadratic equation** is a polynomial with a second degree term as the highest degree term. It is called an **equation of degree 2**".
- The general form a quadratic equation is $ax^2 + bx + c = 0$, where **a**, **b** and **c** are known values except that **a** $\neq 0$ and **x** is the variable or the unknown
- A quadratic equation is said to be a **complete quadratic** if it contains both the second power (**a**), the first power of the variable or unknown quantity (**b**) and the constant term (**c**).
- An **incomplete or pure quadratic** is one in which **b** or **c** (or both) is zero. The only letter that cannot be zero is **a**. If **a** is zero, the equation would not be a quadratic.
- **The Zero Product Property or Null Factor Law** states that if the product of two numbers is zero, then one or both of the numbers is zero.
- **To solve quadratic equations by factoring:**
 - ❖ Simplify each side of the quadratic equation if needed. This would involve removing grouping symbols, clearing fractions, collecting like terms, etc.
 - ❖ Write the quadratic equation in general form, $ax^2 + bx + c = 0$, if needed.
 - ❖ Factor the left side of the equation.
 - ❖ Use the Zero Product Law.
 - ❖ Solve the resulting equations.
- **The quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a \neq 0)$$

- **To graph a quadratic equation $y = ax^2 + bx + c$**
 - ❖ Determine whether the graph opens up ($a > 0$) or down ($a < 0$).
 - ❖ Find the vertex and axis of symmetry using the fact that the x-coordinate of the vertex is $-\frac{b}{2a}$ and the axis of symmetry is $x = -\frac{b}{2a}$.
 - ❖ The y-coordinate of the vertex is found by evaluating substituting the x- value into the original equation.
 - ❖ If the vertex is located above the x-axis and the graph opens down, or if the vertex is located below the x-axis and opens up, find the x-intercepts by solving the equation $ax^2 + bx + c = 0$.
 - ❖ Plot the vertex and the x-intercepts (or two other points on either side of the line of symmetry if there are no x-intercepts) and sketch the parabola.

REVISE LESSONS 13 TO 18. THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 4.

ANSWERS TO PRACTICE EXERCISES 13-18

Practice Exercise 13

1. a. Yes b. No c. No d. Yes e. No
f. Yes g. No h. Yes i. Yes j. No
 2. a. General form: $2x^2 - x - 4 = 0$
a = 2, b = -1, c = -4
b. General form: $x^2 + 3x - 7 = 0$
a = 1, b = 3, c = -7
c. General form: $5x^2 - 11x + 4 = 0$
a = 5, b = -11, c = 4
d. General form: $5x^2 - 3x - 11 = 0$
a = 5, b = -3, c = -11
e. General form: $11x^2 - 6x = 0$
a = 11, b = -6, c = 0
f. General form: $4x^2 - 30x - 3 = 0$
a = 4, b = -30, c = -3
g. General form: $6x^2 - 6x - 3 = 0$
a = 6, b = -6, c = -3
h. General form: $8x^2 - 0x - 5 = 0$
a = 8, b = 0, c = -5
i. General form: $3x^2 - 4x + 12 = 0$
a = 3, b = -4, c = 12
j. General form: $x^2 - x + 1 = 0$
a = 1, b = -1, c = 1
-

Practice Exercise 14

1. a = -1 and a = -6
 2. $x = -\frac{3}{2}$ and $x = \frac{5}{6}$
 3. a = 0 and a = -10
 4. x = -2 and x = -6
 5. $x = -\frac{1}{2}$ and x = 5
 6. $y = \frac{2}{3}$ and $y = -\frac{5}{3}$
 7. x = -7 and x = 2
 8. $x = \frac{2}{5}$ and $x = -\frac{2}{5}$
 9. 5 cm
 10. 7
-

Practice Exercise 15

1. $x = \pm 2$
2. $x = \pm 5\sqrt{2}$
3. $x = \pm 5$
4. $x = 2 \pm 2\sqrt{3}$
5. $x = \pm 5$
6. $x = \pm \sqrt{3}$

Practice Exercise 16

1. $x = -1$ and $x = -5$
2. $x = 4$ and $x = -2$
3. $y = 2$ and $y = -5$

4. $-8 \pm \sqrt{14}$
5. $x = \frac{1}{2}$ and $x = -2$
6. $x = \pm \sqrt{10}$

Practice Exercise 17

1. $x = 3, x = 5$

4. $x = 0, x = -\frac{3}{4}$

2. $x = 5, x = -5$

5. $x = \frac{1 \pm \sqrt{21}}{10}$

3. $x = -\frac{1}{3}, x = -4$

6. $x = \frac{3 \pm \sqrt{41}}{4}$

Practice Exercise 18

1. (a) 9 (b)
- -4
- (c) 0

2. (a)

x	-3	-2	-1	0	1	2	3
$y = 3x^2$	27	12	3	0	3	12	27

- (b)

x	-9	-6	-3	0	3	6	9
$y = \frac{1}{3}x^2$	27	12	3	0	3	12	27

- (c)

x	-4	-3	-2	-1	0	1	2
$y = -10x^2$	-160	-90	-40	-10	0	-10	-40

- (d)

x	-3	-2	-1	0	1	2	3
$3x^2$	27	12	3	0	3	12	27
-3	-3	-3	-3	-3	-3	-3	-3
$y = 3x^2 - 3$	24	8	0	-3	0	8	24

- (e)

x	-6	-4	-2	0	2	4	6
x^2	36	16	4	0	4	16	36
$-2x$	12	8	4	0	-4	-8	-12
$y = x^2 - 2x$	24	8	0	0	0	8	24

3. a. (i) (0.4) (ii) (0, 0) (iii) (0, -2)

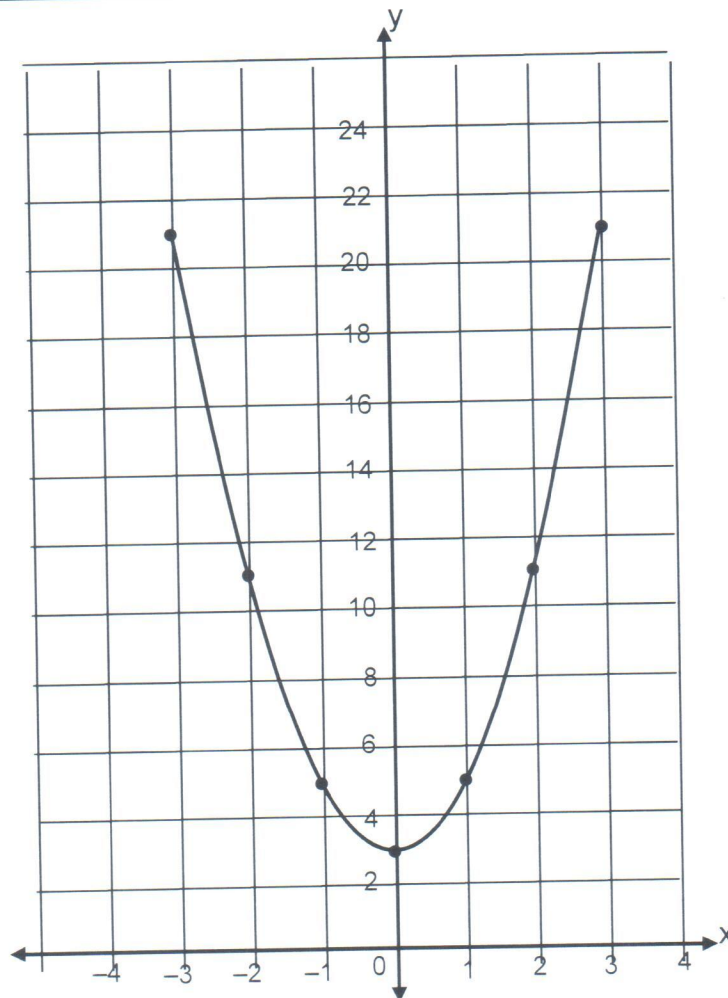
b. $x = 0$

c. (i) -2 (ii) 4

4. a. (3, -4) b. $y = 5$ c. (1, 0) and (5, 0) d. $x = 3$

5.

x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
+3	3	3	3	3	3	3	3
$y=2x^2+3$	21	11	5	3	5	11	21



(b) $y = 3$

(c) (0, 3)

(d) $y = 2x^2 + 3$

END OF TOPIC 3

TOPIC 4

SYSTEMS OF EQUATIONS

Lesson 19:	Solutions of Simultaneous Equations by Graphing
Lesson 20:	Solutions of Simultaneous Equations Using Elimination
Lesson 21:	Solutions of Simultaneous Equations by Substitution
Lesson 22:	Using System of Equation to Solve Word Problems

TOPIC 4: SYSTEM OF EQUATIONS

Introduction



You have learnt how to solve and draw the graphs of linear and quadratic equations in the previous topics.

In this topic, you will learn about systems of linear equations.

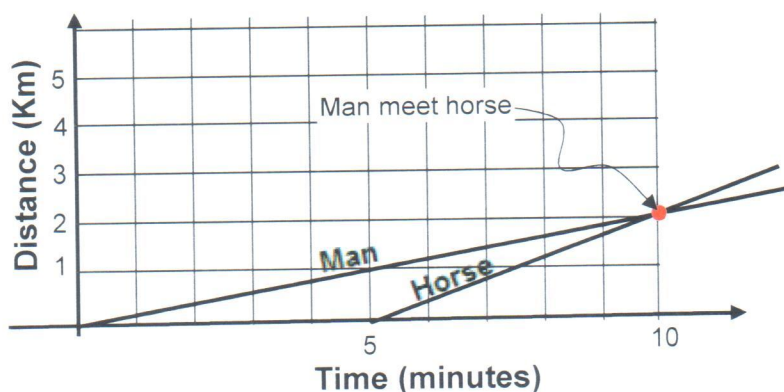
Look at the example of a race between a man and a horse. The man can run 0.2 km every minute and the horse can run 0.5 km every minute. But it takes 6 minutes to saddle the horse.

How far can the man get before the horse catches him?

To answer the question above, we can make two equations and draw the graph.

$$\begin{aligned}\text{Man: } D &= 0.2t \\ \text{The Horse: } D &= 0.5(t - 6)\end{aligned}$$

So we have a system of equations and they are linear as shown below.



In this topic you will learn different methods for solving systems including elimination substitution and graphing.

This topic will teach you different ways of dealing with problems relating to systems of equations in two or more variables and how to recognize systems with no or infinite solutions both algebraically and with graphs.

Lesson 19: Solutions of Simultaneous Equations by Graphing



You have learnt how to solve linear and quadratic equations in the previous topics. You also have learnt how to draw their graphs.



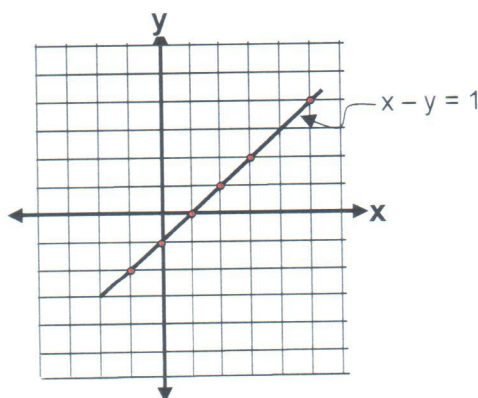
In this lesson, you will:

- define system of equations and give examples
- solve system of equations by graphing
- determine whether a system of equation is consistent, inconsistent or dependent

In Topic 2, we saw that linear equation in two variables x and y , such as $x - y - 1 = 0$, has many solutions which are ordered pairs of numbers making the equation true. Some of the solutions to this equation are

$(0, -1)$, $(1, 0)$, $(-1, -2)$, $(2, 1)$, and $(5, 4)$

The graph of the equation is a straight line, every point of which corresponds to a solution to the equation as shown below.



Suppose that we have a second linear equation, $2x + y - 5 = 0$.

This equation too has infinitely many solutions, some of which are

$(0, 5)$, $(1, 3)$, $(-1, 7)$, $(2, 1)$, and $(-3, 11)$.

A system of two linear equations is a pair of linear equations such as the following:

$$\begin{aligned}x - y - 1 &= 0 \\2x + y - 5 &= 0\end{aligned}$$

We usually refer to such a pair as simply a **system of equations**.

A **solution to a system of equations** is an ordered pair of numbers which is a solution to **both** equations.

For our particular example, since (2, 1) is listed as a solution to both equations, it is a solution to the system of equations. We can check this by substituting (2, 1) into each of the equations.

$$\begin{aligned}\text{For } x - y - 1 &= 0 \\ 2 - 1 - 1 &= 0 \\ 0 &= 0\end{aligned}$$

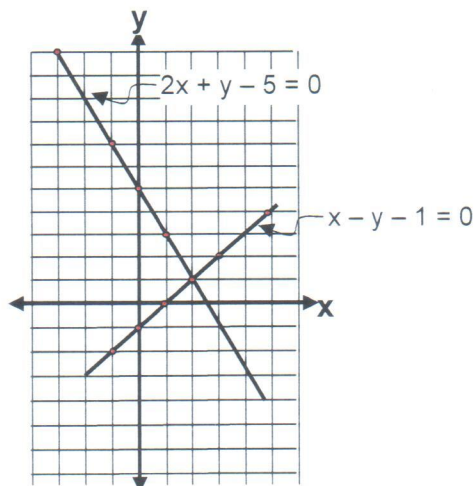
$$\begin{aligned}\text{For } 2x + y - 5 &= 0 \\ 2(2) + 1 - 5 &= 0 \\ 4 + 1 - 5 &= 0 \\ 0 &= 0\end{aligned}$$

If we write both equations in slope-intercept form,

$$\begin{aligned}x - y - 1 &= 0 \\ -y &= -x + 1 \\ y &= x - 1 \\ \text{Slope}(m) &= 1\end{aligned}$$

$$\begin{aligned}2x + y - 5 &= 0 \\ y &= -2x + 5 \\ \text{Slope}(m) &= -2\end{aligned}$$

we can see that the slopes are unequal so that the graphs of the equations are intersecting lines. Graphing both equations in the same Cartesian coordinate axes below we see that the point of intersection corresponds to the solution of the system, (2, 1).



This illustrates the **graphing method** for solving **system of equation**.

This method involves plotting the graphs of the equations, estimating the point of intersection (if such point exists), and verifying by substitution that the point does indeed satisfy each equation.

Example 1

Solve the system using the graphing method.

$$\begin{aligned}x + y &= 2 \\ 2x - y &= -5\end{aligned}$$

Solution: We graph each equation by finding its intercepts.

For equation 1: $x + y = 2$: If $x = 0$, $y = 2$

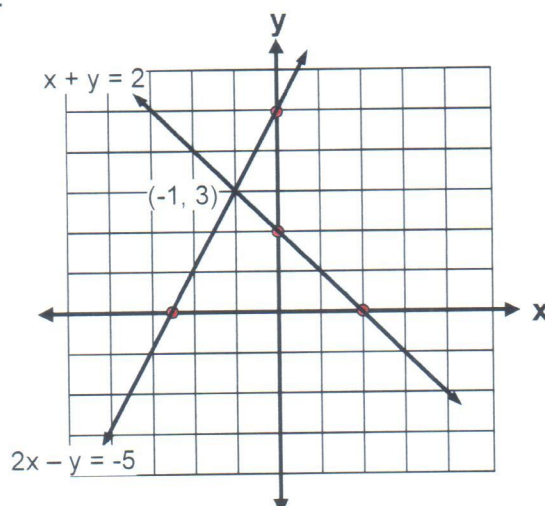
If $y = 0$, $x = 2$

x	y
0	2
2	0

For equation 2: $2x - y = -5$: if $x = 0$, $y = 5$
 If $y = 0$, $x = -\frac{5}{2}$

x	y
0	5
$-\frac{5}{2}$	0

Now, we plot the intercepts.



The point of intersection of the two lines, graphed above appears to have coordinates $(-1, 3)$. We checked by substitution in both equations.

Check:

$$\begin{aligned} \text{In } x + y &= 2 \\ -1 + 3 &= 2 \\ 2 &= 2 \end{aligned}$$

$$\begin{aligned} \text{In } 2x - y &= -5 \\ 2(-1) - 3 &= -5 \\ -5 &= -5 \end{aligned}$$

Thus, the solution to the system is $(-1, 3)$.

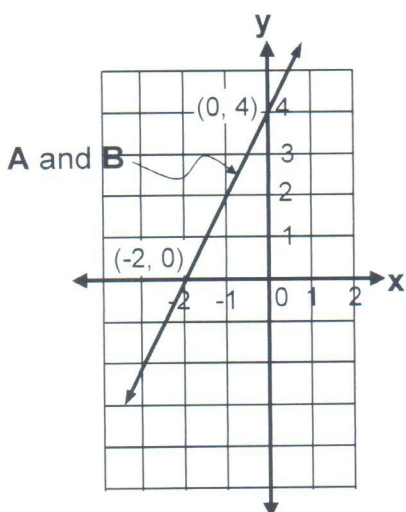
When each of the two linear equations in a system is graphed in the same Cartesian coordinate system, one of the following possibilities occurs:

1. The lines coincide (the equations represent the same line), and the system of equation has **infinitely many solutions**. In this case the system is said to be **dependent**.
2. The lines are parallel (do not intersect at any point), and the system of equations has **no solution**. The system is said to be **inconsistent**.
3. The lines intersect in exactly one point, and the system of equations has **exactly one solution**. The system is said to be consistent and independent.

As examples of these three cases, consider the following examples of three systems of equations and their corresponding graphs on the next page.

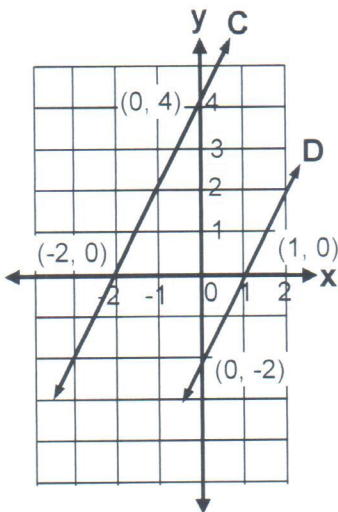
Here are the three systems.

1. $2x - y + 4 = 0 \Rightarrow \mathbf{A}$
 $6x - 3y + 12 = 0 \Rightarrow \mathbf{B}$



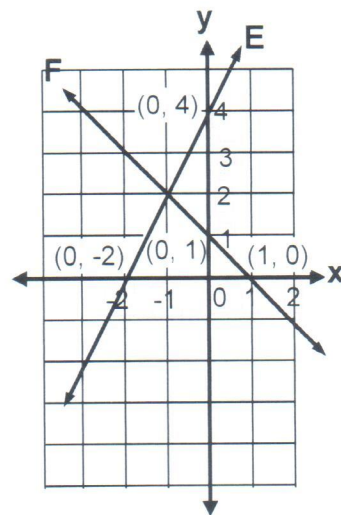
Lines coincide

2. $2x - y + 4 = 0 \Rightarrow \mathbf{C}$
 $2x - y - 2 = 0 \Rightarrow \mathbf{D}$



Lines are parallel

3. $2x - y + 4 = 0 \Rightarrow \mathbf{E}$
 $x + y - 1 = 0 \Rightarrow \mathbf{F}$



Lines intersect at exactly one point

In the first system, lines A and B coincide, in the second system lines C and D are parallel, and in the third, lines E and F intersect at one point.

Let us write each equation in slope –intercept form (solve the equation for y).

$\mathbf{A} \ y = 2x + 4$
 $\mathbf{B} \ y = 2x + 4$

$\mathbf{C} \ y = 2x + 4$
 $\mathbf{D} \ y = 2x - 2$

$\mathbf{E} \ y = 2x + 4$
 $\mathbf{F} \ y = -x + 1$

Recall that in slope –intercept form, the coefficient of x, is the slope and the constant term determines the y-intercept. Also, parallel lines will always have the same slope (or both will have no slope). This information will help us determine the number of solutions to a system of equations.

To determine the number of solutions to a system of equations

1. Write each equation in slope–intercept form and identify the slope and y-intercept.
2. If the slopes are equal and the y-intercepts are equal, the graphs coincide (the equations are equivalent) and the system has **infinitely many solutions**.
3. If the slopes are equal and the –intercepts are unequal, the graphs are parallel and the system has **no solution**.
4. If the slopes are unequal, the graphs intersect in exactly one point and the system has **exactly one solution**.

Example 2

Determine the number of solution of the system: $3x - 2y + 1 = 0$
 $6x - 4y = -2$

Solution:

Solve each equation for y:

$$\begin{aligned} -2y &= -3x + 1 \\ y &= \frac{3}{2}x + \frac{1}{2} \end{aligned}$$

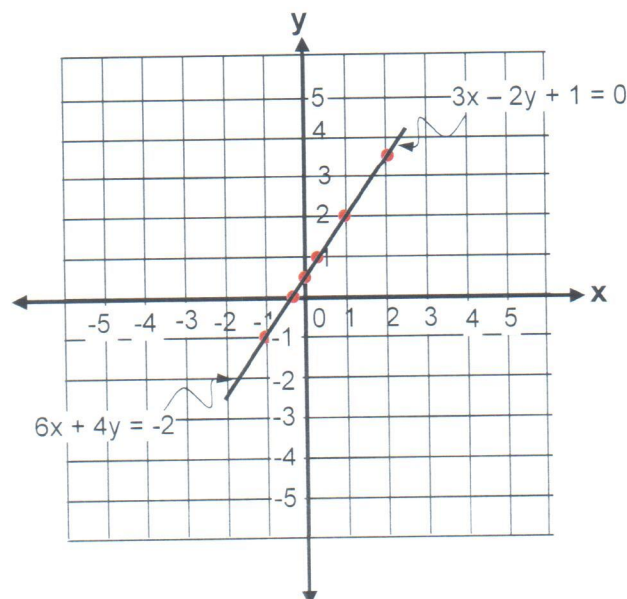
$$\begin{aligned} -4y &= -6x - 2 \\ y &= \frac{3}{2}x + \frac{1}{2} \end{aligned}$$

Since the slopes and y intercepts are equal, the lines **coincide**, and the system has **infinitely many solutions**. That is, any solution to one equation also solves the other, hence is a solution to the system.

Now let us draw the graph of the system.

To draw the graph of the system, make the table of values and plot them on the same coordinate axes. Both the equations in the system have the table of values below which when plotted resulted to one straight line.

x	0	$-\frac{1}{3}$	1	$\frac{1}{3}$	-1	2	$-1\frac{2}{3}$
y	$\frac{1}{2}$	0	2	1	-1	$3\frac{1}{2}$	-2

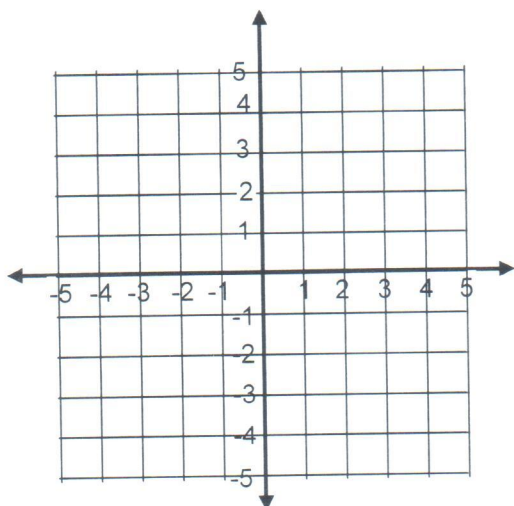


NOW DO THE PRACTICE EXERCISE 19

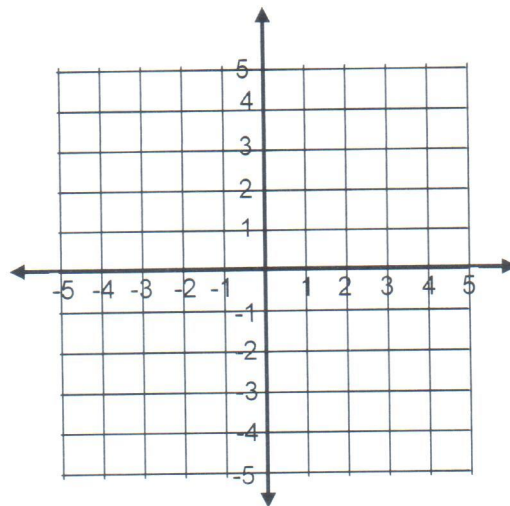
**Practice Exercise 19**

Solve each system by graphing.

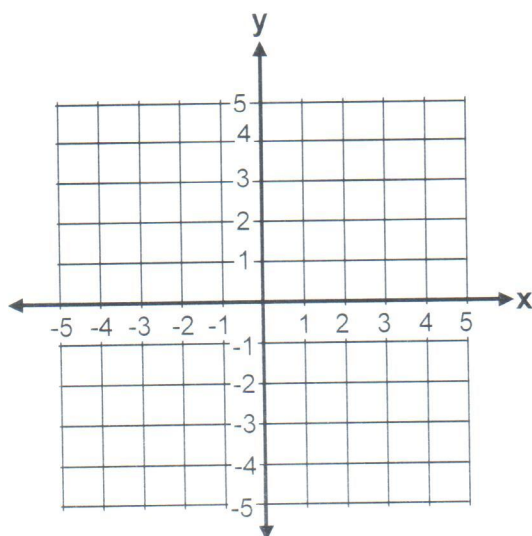
1. $y = 4x + 3$
 $y = -x - 2$



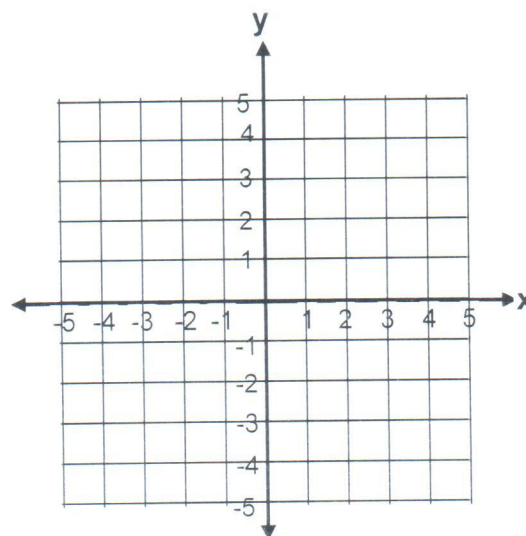
2. $y = -2x + 2$
 $y = -2x - 2$



3. $y = -1$
 $y = -\frac{5}{2}x + 4$

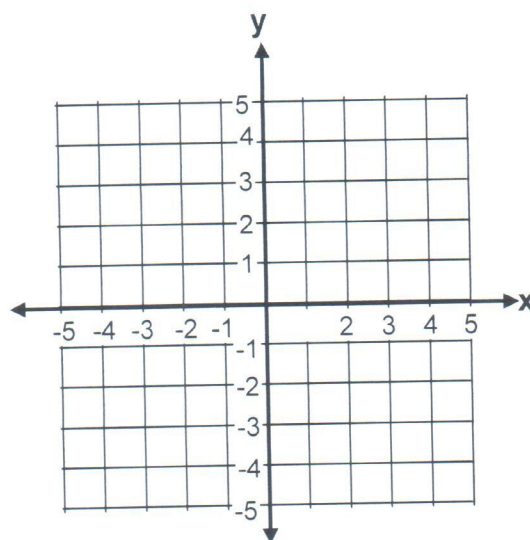
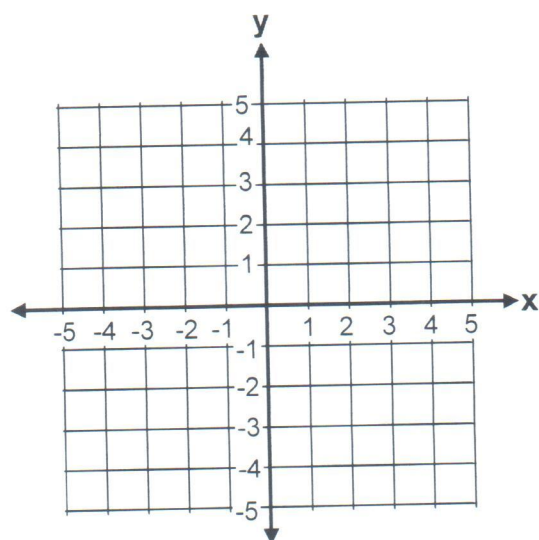


4. $2x - y = 5$
 $x + 4y = 7$



5. $3x + 5y = 15$
 $6x + 10 = 30$

6. $x + 2y = 0$
 $2x - y = 0$



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 20: Solutions of Simultaneous Equations Using Elimination



You have learned the meaning of a system of equations. You have also learned to identify a system of equations as linear and quadratic system.



In this lesson, you will:

- solve system of linear equations by using the elimination method.

One of the methods of solving systems of equations is the **addition-subtraction method** also called the **method of elimination**.

"Eliminate" means to **remove**: this method works by removing variables until there is just one left.

The elimination method is similar to the method you probably learned for solving simple equations.

For example, if we had the equation $x + 6 = 11$, we would write "-6" under either side of the equation, and then we "add down" to get " $x = 5$ " as the solution.

$$\begin{array}{r} x + 6 = 11 \\ -6 = -6 \\ \hline x = 5 \end{array}$$

We will do something similar with the elimination method.

Example 1

Solve the following system of equations using elimination.

$$\begin{array}{r} x + y = 4 \\ 2x - y = 5 \end{array}$$

Solution:

You see that $2x - y$ and 5 are both names or expressions for the same number. Therefore we can add $2x - y$ to the left side and 5 to the right side to obtain another equation with the same solution.

$$\begin{array}{r} (x + y) + (2x - y) = 4 + 5 \\ 3x = 9 \end{array}$$

Generally, we add vertically and avoid the intermediate steps.

$$\begin{array}{r} x + y = 4 \\ 2x - y = 5 \\ \hline 3x = 9 \\ x = 3 \end{array}$$

By adding the two equations, we “eliminate” the y term and obtain one equation in one variable. Once we know that x is 3, we substitute this value into either of the two original equations to find the value of y .

$$\begin{aligned}\text{Substituting in the first equation, } x + y &= 4 \\ 3 + y &= 4 \\ y &= 1\end{aligned}$$

It does not matter which equation we use for solving for y . We will get the same answer either way. If we use the second equation we have:

$$\begin{aligned}2x - y &= 5 \\ 2(3) - y &= 5 \\ 6 - y &= 5 \\ -y &= -1 \\ y &= 1\end{aligned}$$

Thus the solution is **(3, 1)**. Check this by substituting in both of the original equations

Example 2

Solve the following system using elimination.

$$\begin{aligned}2x + y &= 9 \\ 3x - y &= 16\end{aligned}$$

Solution:

Notice that, if we add down the two equations, the y 's will cancel out. So

$$\begin{aligned}2x + y &= 9 \\ 3x - y &= 16 \\ \hline 5x &= 25 \\ x &= 5\end{aligned}$$

Now we can solve for y , by substituting $x = 5$ into either of the two original equations.

$$\begin{aligned}\text{In the first equation: } 2x + y &= 9 \\ 2(5) + y &= 9 \\ 10 + y &= 9 \\ y &= -1\end{aligned}$$

$$\begin{aligned}\text{In the second equation: } 3x - y &= 16 \\ 3(5) - y &= 16 \\ 15 - y &= 16 \\ y &= -1\end{aligned}$$

Thus the solution is **(5, -1)**. Check this by substituting in both of the original equations.

Example 3

Solve the following system using elimination.

$$\begin{aligned}x - 2y &= -9 \\x + 3y &= 16\end{aligned}$$

Solution:

Notice that the x terms would cancel out if only they have opposite signs. We can create cancellation by subtracting the second equation from the first equation.

$$\begin{aligned}x - 2y &= -9 \\x + 3y &= 16 \\ \hline -5y &= -25 \\ y &= 5\end{aligned}$$

Now we can solve for y , by substituting $y = 5$ into either of the two original equations.

$$\begin{aligned}\text{In the first equation: } x - 2(5) &= -9 \\ x - 10 &= -9 \\ x &= 1\end{aligned}$$

$$\begin{aligned}\text{In the second equation: } x + 3(5) &= 16 \\ x + 15 &= 16 \\ x &= 1\end{aligned}$$

Thus the solution is **(1, 5)**. Check this by substituting in both of the original equations.

At times, addition or subtraction alone will not eliminate a variable and yield one equation in only one variable. When this happens, we may have to multiply one or both equations by a number that makes the coefficients of one variable negative of each other. Then by adding we eliminate that variable.

Let us look at the following example, to understand this better.

Example 4

Solve the following system of equations using elimination.

$$\begin{aligned}2x - y &= 9 \\3x + 4y &= -14\end{aligned}$$

Solution:

By adding or subtracting the two equations as given, we do not eliminate a variable. However, if we multiply both sides of the first equation by 4 (using the multiplication rule), the resulting equation is

$$8x - 4y = 36$$

Add this equation to the second equation to solve for x.

$$\begin{array}{r} 8x - 4y = 36 \\ 3x + 4y = -14 \\ \hline 11x = 22 \\ x = 2 \end{array}$$

To solve for y, substitute $x = 2$ into either of the two original equations.

In first equation:	$2x - y = 9$	In the second equation:	$3x + 4y = -14$
	$2(2) - y = 9$		$3(2) + 4y = -14$
	$4 - y = 9$		$6 + 4y = -14$
	$y = -5$		$4y = -20$
			$y = -5$

Thus the solution is **(2, -5)**. Check by substituting in both of the original equations.

Example 5

Solve the system by elimination:

$$\begin{array}{r} 3x + 5y = -2 \\ 5x + 3y = 2 \end{array}$$

Solution:

Multiply the first equation by 5 and the second equation by 3, making both coefficients of the x terms, 15.

$$\begin{array}{r} (3x + 5y = -2) \cdot 5 \rightarrow 15x + 25y = -10 \\ (5x + 3y = 2) \cdot 3 \rightarrow 15x + 9y = 6 \end{array}$$

$$\begin{array}{r} 15x + 25y = -10 \\ 15x + 9y = 6 \\ \hline 16y = -16 \\ y = -1 \end{array}$$

Subtract the second equation from the first to eliminate x.

Substitute $y = -1$ into either the two original equation to find x.

In first equation:

$$\begin{array}{r} 3x + 5y = -2 \\ 3x + 5(-1) = -2 \\ 3x - 5 = -2 \\ 3x = 3 \\ x = 1 \end{array}$$

In the second equation:

$$\begin{array}{r} 5x + 3y = 2 \\ 5x + 3(-1) = 2 \\ 5x - 3 = 2 \\ 5x = 5 \\ x = 1 \end{array}$$

Thus the solution is **(1, -1)**. Check by substituting in both of the original equations.

We can now summarize the steps we have learned.

To solve a system of equations using Elimination by Addition or Subtraction

1. Apply the multiplication rule. If necessary multiply either or both of the given equations by numbers which will make the coefficients of one of the variables the same or alike in both equations.
2. If these same coefficients have like signs, subtract one equation from the other; if they have unlike signs, add the equations to eliminate the variable or letter whose coefficients were made the same.
3. The result will be an equation in one variable. Solve the equation.
4. Substitute the value thus obtained in one of the given equations to obtain the value of the other unknown.
5. Check in the original equations.

NOW DO PRACTICE EXERCISE 20

**Practice Exercise 20**

Solve each system of equation by elimination.

1. $x - y = 11$
 $2x + y = 19$

Answer: _____

2. $8x + y = -16$
 $-3x + y = -5$

Answer: _____

3. $-2x - 9y = -25$
 $-4x - 9y = -23$

Answer: _____

4. $7x + 2y = 24$
 $8x + 2y = 30$

Answer: _____

5. $x + 2 = 0$
 $3x + 2y = 10$

Answer: _____

6. $x + 2y = -9$
 $3x + 4y = -17$

Answer: _____

7. $5x + y = 9$
 $10x - 7y = -18$

Answer: _____

8. $-4x + 9y = 9$
 $x - 3y = -6$

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 21: Solution of Simultaneous Equations by Substitution



You have learnt how to solve system of equations using the elimination method in the previous lesson.



In this lesson, you will:

- learn the steps in solving system of equation using substitution method.
- solve system of equations by using substitution method.

Another method of solving a simultaneous system of equations is by **substitution**.

It is important that you learn the substitution method of solving simple system of equations so that you can apply this method to solve more complicated systems in your further study of mathematics.

In this method, we start with one equation in the system and solve for one variable in terms of the other variable.

Example 1

Solve the system: $x - 2y = 11 \longrightarrow$ Equation [1]
 $3x + 5y = -11 \longrightarrow$ Equation [2]

Solution:

Solve Equation [1] for x : $x - 2y = 11$
 $x = 11 + 2y$

Substitute $11 + 2y$ in place of x in Equation [2]:

$$\begin{aligned} \text{Equation 2:} \quad & 3x + 5y = -11 \\ & 3(11 + 2y) + 5y = -11 \\ & 33 + 6y + 5y = -11 \\ & 33 + 11y = -11 \\ & 11y = -11 - 33 \\ & 11y = -44 \\ & \mathbf{y = -4} \end{aligned}$$

Substitute $y = -4$ in $x = 11 + 2y$, to find the value of x .

$$\begin{aligned} x &= 11 + 2y \\ x &= 11 + 2(-4) \\ x &= 11 - 8 \\ \mathbf{x} &= \mathbf{3} \end{aligned}$$

Therefore, $(3, 4)$ is the solution for the system.

To check: Substitute $x = 3$ and $y = -4$ in any of the two equations.

In Equation [1]: $x - 2y = 11$
 $3 - 2(-4) = 11$
 $3 + 8 = 11$
 $11 = 11$ True

In Equation [2]: $3x + 5y = -11$
 $3(3) + 5(-4) = -11$
 $9 - 20 = -11$
 $-11 = -11$ True

Therefore $(3, -4)$ is a solution.

How to choose which variable or letter to solve for

- a. One of the equations may already be solved for a variable or letter.

For example the system: $2x - 5y = 4$

$$y = 3x + 7$$

Already solved for y

- b. One of the variables or letters may have a coefficient of 1.

For example the system: $2x + 6y = 3$

$$x - 4y = 2 \Rightarrow x = 4y + 2$$

x has a coefficient of 1

- c. Choose the variable or letter with the smallest coefficient

For example the system: $11x - 7y = 10$

$$14x + 2y = 9 \Rightarrow y = \frac{9 - 14x}{2}$$

Smallest of the four coefficients

Smallest possible denominator in this case

Now study the examples.

Example 2

Solve the system: $3x + 10y = 2 \Rightarrow$ Equation [1]

$x - 2y = 6 \Rightarrow$ Equation [2]

Solution:

Solve for x in terms of y in Equation [2]. Thus we have $x = 6 + 2y$

Substitute this value in Equation [1].

We get

$$\begin{aligned}
 3x + 10y &= 2 \\
 3(6 + 2y) + 10y &= 2 \\
 18 + 6y + 10y &= 2 \\
 18 + 16y &= 2 \\
 16y &= 2 - 18 \\
 16y &= -16 \\
 y &= -1
 \end{aligned}$$

Substitute -1 for y in Equation [2] to find for the value of x.

$$\begin{aligned}
 x - 2y &= 6 \\
 x - 2(-1) &= 6 \\
 x + 2 &= 6 \\
 x &= 6 - 2 \\
 x &= 4
 \end{aligned}$$

To check substitute $x = 4$ and $y = -1$ in any of the two equations.

In Equation [1]:	$3x + 10y = 2$	In Equation 2:	$x - 2y = 6$
	$3(4) + 10(-1) = 2$		$4 - 2(-1) = 6$
	$12 - 10 = 2$		$4 + 2 = 6$
	$2 = 2$ True		$6 = 6$

Therefore, (4, -1) is the solution.

Example 3

Solve the system:

$$\begin{aligned}
 9x + 6y &= 6 \Rightarrow \text{Equation 1} \\
 6x + 4y &= 4 \Rightarrow \text{Equation 2}
 \end{aligned}$$

Smallest of the coefficients

Solution:

From Eq. [2], we solve for y in terms of x.

$$\begin{aligned}
 6x + 4y &= 4 \\
 4y &= 4 - 6x \\
 y &= \frac{4 - 6x}{4} \\
 y &= \frac{2 - 3x}{2} \Rightarrow \text{Equation [3]}
 \end{aligned}$$

Substitute $\frac{2-3x}{2}$ in place of y in Equation [1].

Equation 1: $9x + 6y = 6$

$$9x + 3\cancel{6}\left(\frac{2-3x}{\cancel{2}}\right) = 6$$

$$9x + 3(2 - 3x) = 6$$

$$9x + 6 - 9x = 6$$

$$6 = 6$$

Cancel 6 and 2 on the LHS

Expand

An equality

Therefore, in this case, **any ordered pair** that satisfies Equation 1 or Equation 2 is a solution of the system.

Example 4

Solve the system: $3x - 2y - 5 = 0 \Rightarrow \text{Equation [1]}$
 $2x + 3y - 12 = 0 \Rightarrow \text{Equation [2]}$

Solution: From Eq. [2], we solve for y in terms of x .

$$2x + 3y - 12 = 0$$

$$3y = -2x + 12$$

$$y = \frac{-2x + 12}{3} \Rightarrow \text{Equation [3]}$$

Substitute this for y in Equation [1].

Equation [1]: $3x - 2y - 5 = 0$

$$3x - 2\left(\frac{-2x + 12}{3}\right) - 5 = 0 \quad \text{Substitute}$$

$$9x + 4x - 24 - 15 = 0 \quad \text{Multiply both sides by 3}$$

$$13x - 39 = 0$$

$$13x = 39$$

$$x = 3$$

Substitute $x = 3$ in Equation 3 to find for y .

$$y = \frac{-2x + 12}{3}$$

$$y = \frac{-2(3) + 12}{3}$$

$$y = \frac{-6 + 12}{3}$$

$$y = 2$$

Therefore, the solution is **(3, 2)**.

The method of substitution may be summarized as follows:

1. Solve one of the equations for one of the variables or letters in terms of the other.
2. Substitute this expression obtained in Step 1 the other equation.
3. Solve the resulting equation In Step 2.
4. Substitute the value obtained in Step 3 back into Step 1 for the other variable.

NOW DO THE PRACTICE EXERCISE 21

**Practice Exercise 21**

Solve the following system of equation by using substitution method.

1. $y = 4 - x$
 $2x + y = 5$

Answer: _____

2. $y = x + 2$
 $3x + y = 6$

Answer: _____

3. $y = x + 5$
 $5x - y = -1$

Answer: _____

4. $y = x + 2$
 $3x + 2y = -1$

Answer: _____

5. $8x - 5y = 4$
 $2x - 3y = -20$

Answer: _____

6. $8x + 4y = 7$
 $3x + 6y = 6$

Answer: _____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 22: Using System of Equation to Solve Word Problems



You have learned to solve system of equation using the method of substitution in the previous lesson.



In this lesson, you will:

- solve word problems using system of equations .

In solving word problems that involve more than one unknown, it is sometimes difficult to represent each unknown in terms of a single letter. In this section we eliminate that difficulty using a different letter for each unknown.

There are different real life problems that can be solved using system of linear equations.

Example 1

The sum of two numbers is 20. Their difference is 6. What are the numbers?

Solution: Let x = larger number
 y = smaller number

The sum of two numbers is 20.
 $\underbrace{\hspace{2cm}}_{x + y} \quad \underbrace{\hspace{1cm}}_{= 20}$

Their difference is 6.
 $\underbrace{\hspace{2cm}}_{x - y} \quad \underbrace{\hspace{1cm}}_{= 6}$

Using elimination by addition and subtraction method

$$x + y = 20 \Rightarrow \text{equation 1}$$

$$\underline{x - y = 6} \Rightarrow \text{equation 2}$$

$$2x = 26$$

$$x = 13 \text{ larger number}$$

Substitute $x = 13$ in Equation 1 to find y .

$$x + y = 20$$

$$13 + y = 20$$

$$y = 20 - 13$$

$$y = 7 \text{ smaller number}$$

Check:

In equation 1: $x + y = 20$

$$13 + 7 = 20$$

$$20 = 20$$

In Equation 2: $x - y = 6$

$$13 - 7 = 6$$

$$6 = 6$$

Therefore, the numbers are 13 and 6.

Example 2

2000 tickets were sold in an exhibition at Vision City. The cost of a ticket for an adult is K4 and for a child is K2. The total amount collected was K6400. Find the number of adults and child tickets sold.

Solution:

Step 1: Identify the given information and what is being asked.

Total number of tickets sold = 2000

Cost of ticket for an adult = K4

Cost of ticket for a child = K2

Total amount collected = K6400

We are asked to find the number of adult tickets and children tickets sold.

Let x be the no. of adults tickets sold

y be the no. of child tickets sold

Step 2: Form the equations
Total tickets sold = 2000

$$\underbrace{\text{No. of adults tickets sold}}_x + \underbrace{\text{No. of child tickets sold}}_y = \underbrace{\text{Total tickets sold}}_{2000}$$

Total amount collected = K6400

Since there are x adult tickets sold and the cost is K4 each,

Amount collected from adult tickets = $4x$

Since there are y child tickets sold and the cost is K2 each,

Amount collected from child tickets = $2y$

$$\underbrace{\text{Amt. collected from adult}}_{4x} + \underbrace{\text{Amt. collected from children}}_{2y} = \underbrace{\text{Total amount collected}}_{6400}$$

Now we have a system of two equations.

$$x + y = 2000 \Rightarrow \text{Equation 1}$$

$$4x + 2y = 6400 \Rightarrow \text{Equation 2}$$

Step 3: Solve the system to find for the value of x and y .

Multiply Equation 1 with -2 . and add the resulting equation and Equation 2 to eliminate the variable y .

$$-2(x + y = 2000) = -2x - 2y = -4000$$

$$-2x - 2y = -4000$$

$$\underline{4x + 2y = 6400}$$

$$2x = 2400 \quad \text{Divide both sides by 2 and simplify.}$$

$$x = 1200$$

Step 4: Substitute the value of $x = 1200$ in either of the two equations.

In equation 1: $x + y = 2000$

$$1200 + y = 2000$$

$$y = 2000 - 1200 \quad \text{Subtract 1200 from both sides}$$

$$y = 800$$

Step 5:

So the solution for the given system of equations is **(1200, 800)**, which means **1200 adult tickets** and **800 child tickets** were sold at Vision City.

The above problem can also be solved by substitution method since the coefficients of x and y in Equation 1 is one.

NOW DO THE PRACTICE EXERCISE 22
--

**Practice Exercise 22**

Solve the following word problems.

- 1) Find the value of two numbers if their sum is 12 and their difference is 4.
-

- 2) The difference of two numbers is 3. Their sum is 13. Find the numbers.
-

- 3) Flying to Kokopo with a tailwind a plane averaged 158 km per hour. On the return trip the plane only averaged 112 km per hour while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.
-

- 4) The school that Kila goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of K38. The school took in K52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.
-

- 5) The sum of the digits of a certain two-digit number is 7. Reversing its digits increases the number by 9. What is the number?
-

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.
--

TOPIC 4: SUMMARY

This summarizes some of the important ideas and concepts to remember.

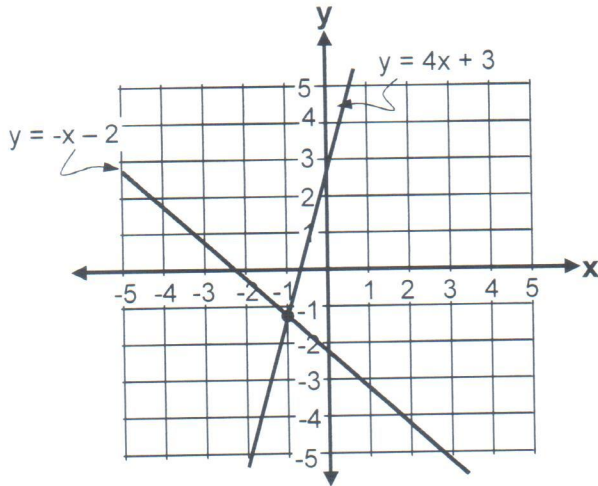
- A **system of equations** is a collection of two or more **equations** with a same set of unknowns.
- A **solution to a system of equations** is an ordered pair of numbers which is a solution to both equations.
- **Graphical methods of solving a system of equations** involves plotting the graphs of the equations, estimating the point of intersection and verifying by substitution that the point satisfies each equation.
- The system of equations is said to be **dependent** when it has **infinitely many solutions** and the lines **coincide**.
- The system of equations is said to be **inconsistent** when the lines are **parallel** and the system has **no solution**.
- When the lines **intersect in exactly one point** and the system of equations has **exactly one solution**, it is said to be **consistent** and **independent**.
- To determine the number of solutions to a system of equations:
 - Write the equation in slope-intercept form and identify the slope and y-intercept.
 - If the slopes are equal and the y-intercepts are equal, the graphs coincide the system has **infinitely many solutions**
 - If the slopes are equal and the y-intercepts are unequal, the graphs are parallel and the system has **no solution**.
 - If the slopes are unequal, the graphs intersect in exactly one point and the system has exactly **one solution**.
- **Elimination method by addition and subtraction** of solving system of equations works by removing the variables until there is just one left.
- **Eliminate** means to **remove**.
- To solve a system of equation by elimination by addition or subtraction:
 - Apply the multiplication rule. If necessary multiply either or both of the given equations by numbers which will make the coefficients of one of the variables the same or alike in both equations.
 - If these same coefficients have like signs, subtract one equation from the other; if they have unlike signs, add the equations to eliminate the variable or letter whose coefficients were made the same.
 - The result will be an equation in one variable. Solve the equation.
 - Substitute the value thus obtained in one of the given equations to obtain the value of the other unknown.
 - Check in the original equations.
- **Substitution method** of solving system of equations starts with one equation in the system and solve for one variable in terms of the other variable.
- The substitution method is summarized as follows:
 - Solve one of the equations for one of the variables or letters in terms of the other.
 - Substitute this expression obtained in Step 1 the other equation.
 - Solve the resulting equation In Step 2.
 - Substitute the value obtained in Step 3 back into Step 1 for the other variable.

REVISE LESSONS 19 TO 22. THEN DO TOPIC TEST 4 IN ASSIGNMENT BOOK 4.

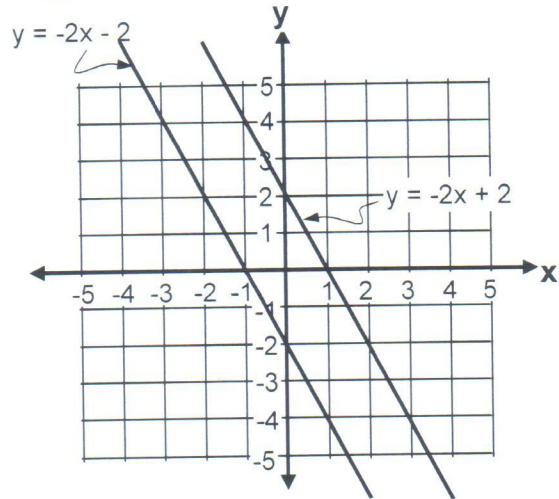
ANSWERS TO PRACTICE EXERCISE 19 – 22

Practice Exercise 19

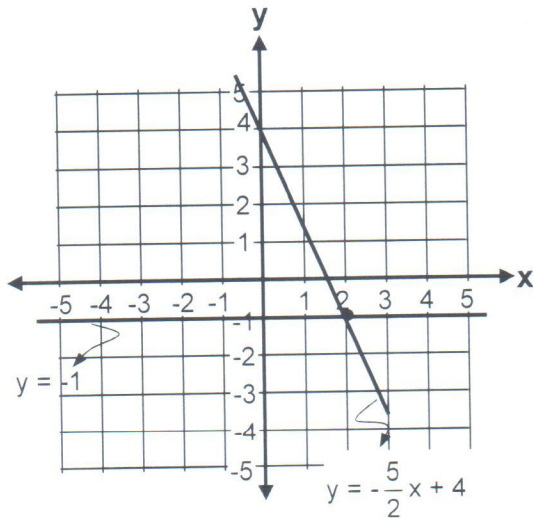
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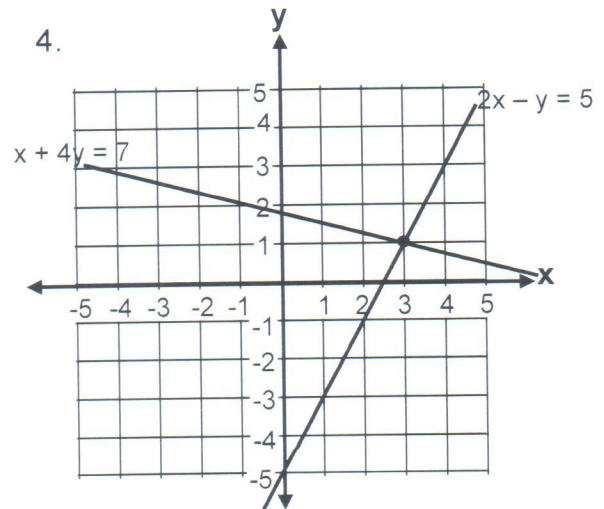
2.



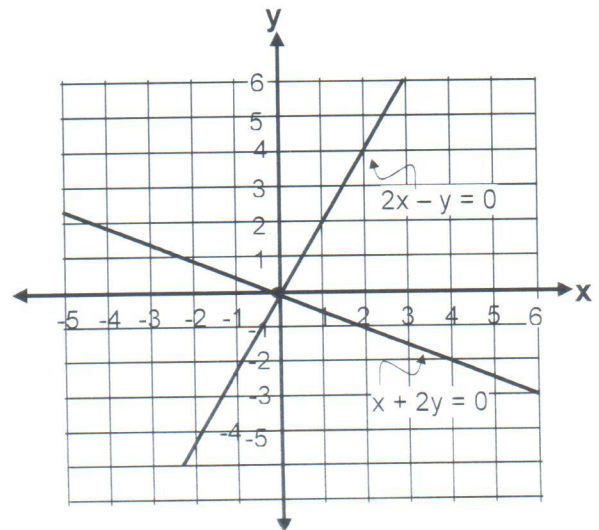
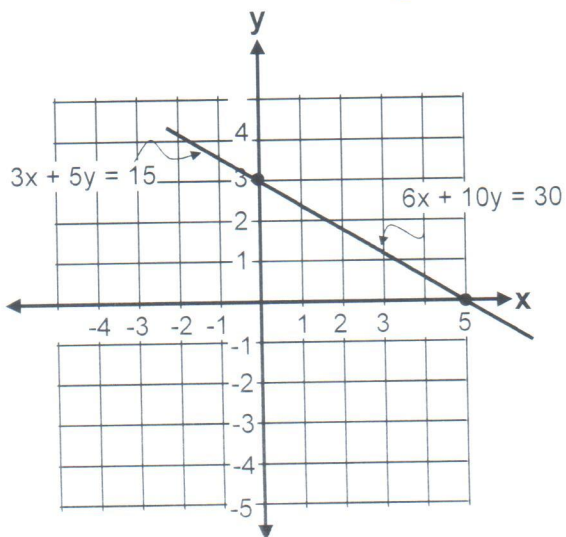
3.



4.



5.



Practice Exercise 20

1. (10, -1)
2. (-1, -8)
3. (-1, 3)
4. (6, -9)
5. (-2, 8)
6. (1, -5)
7. (1, 4)
8. (9, 5)

Practice Exercise 21

1. (1, 3)
2. (1, 3)
3. (1, 6)
4. (-1, 1)
5. (-9, -11)
6. $\left(\frac{1}{2}, \frac{3}{4}\right)$

Practice Exercise 22

1. 4 and 8
2. 5 and 8
3. Plane: 135 km per hour; Wind: 23 km per hour
4. Senior citizen tickets: K8; Child ticket; K14
5. 34

END OF UNIT 4

NOW YOU MUST COMPLETE ASSIGNMENT 4.

REFERENCES

- FODE Grade 10 Formal Mathematics Book Unit 6, Graphs and Statistics, 2008 Edition
 - FODE Grade 10 Formal Mathematics Unit 3, Algebra, 2008 Edition
 - Oxford: Modular Mathematics; Intermediate GCSE for AQA, by: Brian Gaulter and Clive Simpson
 - Pearson: Signpost Mathematics 8, Second Edition by: A. McSeveny, R. Conway and S. Wilkes
 - Intermediate Algebra by Johnston and Willis
 - 2 Units Mathematics Book 1, by S. B. Jones and K.E. Couchman
 - Hienemann VCE Zone, Further Mathematics Enhanced page 269
 - www.algebra-class.com/system-of-equations.html
 - www.purplemath.com/modules/systlin1.htm
 - [www.mathwarehouse.com/algebra/linear equation/systems-of-equation](http://www.mathwarehouse.com/algebra/linear-equation/systems-of-equation)
 - <http://www.study-maths.co.uk/topics/factorisingQuadraticExpressions.php>
 - <http://cdn.kutasoftware.com/Worksheets/Alg1/Systems%20of%20Equations%20Word%20Problems.pdf>
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6	MENDI	P. O. Box 237, Mendi	72228142	77522846	Mr Wari Tange	72229053
7	GOROKA	P. O. Box 990, Goroka	72228116	77522847	Ms Ovin Tuki	72229054
8	KUNDIAWA	P. O. Box 95, Kundiawa	72228144	77522848	Mr Denge Gundu	72229056
9	MT HAGEN	P. O. Box 418, Mt. Hagen	72228148	77522849	Mr Robert Maki	72229057
10	VANIMO	P. O. Box 38, Vanimo	72228140	77522850	Mrs Isabella Danti	72229060
11	WEWAK	P. O. Box 583, Wewak	72228122	77522851	Mr David Wombui	72229062
12	MADANG	P. O. Box 2071, Madang	72228126	77522852	Mrs Applonia Bogg	72229063
13	LAKE	P. O. Box 4969, Lae	72228132	77522853	Ms Cathrine Kila	72229064
14	KIMBE	P. O. Box 328, Kimbe	72228150	77522854	Mrs Bernadette Litom	72229065
15	RABAUL	P. O. Box 83, Kokopo	72228118	77522855	Mrs Verlyn Vavai	72229067
16	KAVIENG	P. O. Box 284, Kavieng	72228136	77522856	Mr John Lasisi	72229069
17	BUKA	P. O. Box 154, Buka	72228108	77522857	Mr Marlyne Meiskamel	72229073
18	MANUS	P. O. Box 41, Lorengau	72228128	77522858	Ms Roslyn Keket	72229080
19	NCD	C/- FODE HQ	72228134	77522859	Mrs Marina Tomiyavau	72229081
20	WABAG	P. O. Box 259, Wabag	72228120	77522860	Mr Salas Kamberan	72229082
21	HELA	P. O. Box 63, Tari	72228141	77522861	Mr Ogai John	72229083
22	JIWAKA	c/- FODE Hagen	72228143	77522862	Joseph Walep	72229085

SUBJECT AND GRADE TO STUDY

Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional